Journal of Monetary Economics 24 (1989) 109–122. North-Holland

TESTING FOR RATIONAL BUBBLES WITH EXOGENOUS OR ENDOGENOUS FUNDAMENTALS
The German Hyperinflation Once More

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Received January 1987, final version received January 1989

The presence of rational price bubbles during the German hyperinflation is tested under two different structural assumptions on the money process. If the money supply is constrained to be exogenous to the current inflation rate, the hypothesis of no bubble can be rejected. However, this is no longer found to be true when a feedback rule from inflation to money creation is allowed. The analysis contradicts previous results presented in the literature.

1. Introduction

It is well known that rational, exploding bubbles can be empirically indistinguishable from misspecification of the market fundamentals solution.[1] This paper provides an additional example in which the conclusion of an empirical test for bubbles is reversed by allowing a general enough specification for the forcing variables.

The episode studied is, once more, the German hyperinflation of the Twenties. The generalization does not come from including nonstationarities in the process followed by the market fundamentals (in our case, for example, a probability of monetary reform), but by taking a more agnostic view on their exogeneity. Specifically, our result is that while the joint hypothesis of no bubble and exogenous money supply can be rejected, this is no longer true when a feedback rule from inflation to money creation is allowed. Hamilton and Whiteman (1985) found that nonstructural tests of joint stationarity of money and prices time series did not support the evidence of a rational bubble. It is encouraging to see that the same conclusion can be reached when

*I thank Oliver Blanchard, Rudi Dornbusch, Jim Powell, Mike Whinston, Jeff Wooldridge, an anonymous referee, the editors of this journal, and especially Jonathan Feinstein and Danny Quah for many helpful comments. Financial support from the Social Sciences Research Council is gratefully acknowledged.

[1] See, for example, the discussion in Flood and Garber (1980a), Hamilton and Whiteman (1985), Hamilton (1986), Diba and Grossman (1988).

we generalize the structural specification to better capture the information we have on the process followed by the fundamentals.

The decision to focus on the possible endogeneity of the money supply derived from the conclusions of the rich empirical literature on the German hyperinflation. The possibility of an expected structural change in the money supply process has been widely studied [Flood and Garber (1980b, 1983), LaHaye (1985)], and the agreement has emerged that such probability was very close to zero until the end of the Summer of 1923. Therefore its neglect would not bias estimates that exclude data from the last months of 1923, as has been customary in papers testing for bubbles. On the other hand, both historical and econometric evidence have repeatedly stressed the likelihood of a feedback rule from inflation to money supply [see, among others, Sargent and Wallace (1973), Frenkel (1976), Sargent (1977), Evans (1978), Feldman (1985), Webb (1985)], but with the only exception of Burmeister and Wall (1987) this assumption has not been rigorously embodied in formal tests for price bubbles.[2]

The main reason for this omission comes from the technical complication that such generalization was believed to require. One of the goals of this paper is to show how the problem can be solved simply and elegantly. Intuitively, a univariate representation for the money process is in general legitimate whether money is exogenous or endogenous, but the correlations between the variables of the model and the residuals will be different in the two cases. This implies different estimation methods, and hence the possibility of distinguishing the two scenarios.

The test performed in this paper was proposed by West (1985) [and independently by Casella (1985)], has been applied in the literature [Meese (1986)], and has been discussed at length in Flood, Hodrick, and Kaplan (1986). West used it to study stock market prices and Meese exchange rates, but it also seems particularly appropriate to the analysis of hyperinflations. It is a specification test, detecting inconsistencies in the estimated parameters when the bubble is excluded. As stressed by Flood, Hodrick, and Kaplan, one of its advantages is to conduct the estimation under the null hypothesis of no bubble, avoiding the difficult issue of characterizing the asymptotic distribution in the presence of the bubble term.

This technical feature, its simplicity, and the possibility to avoid the structural specification of the process followed by the fundamentals are its major strengths, when compared to the estimation in Burmeister and Wall (1987). Contrary to our result, Burmeister and Wall find evidence of a bubble when the rate of money growth is allowed to respond to current expected

[2] Sargent and Wallace (1984) discuss the time series specification of a model with bubbles and endogenous money supply, pointing out the possibility of multiple equilibria. However, they do not present estimation results.

inflation. In a previous work [Burmeister and Wall (1982)] they had reached the same conclusion when money was constrained to be exogenous. This contrasted with Flood and Garber (1980a), the first paper on the topic, which had assumed exogenous money and rejected the null hypothesis of a deterministic bubble.

The next section of the paper describes the model. The empirical results are then presented and evaluated, in the two cases of exogenous (section 3) or endogenous (section 4) money supply. Concluding remarks and proposals for future research are in section 5. A short appendix provides a brief description of the data.

2. The model

Following most studies on hyperinflation, the analysis is centered on a Cagan (1956) money demand equation:

$$m_t - p_t = \beta + \alpha({}_tp_{t+1} - p_t) + \varepsilon_t, \tag{1}$$

where m_t is the log of money at time t, p_t is the log of the price level at time t, $({}_tp_{t+1} - p_t)$ is the expected inflation rate between t and $t+1$ conditional on information known at time t, and ε_t is a stochastic term assumed to follow a random walk:

$$\varepsilon_t = \varepsilon_{t-1} + u_t, \tag{2}$$

where u_t is white noise. This is the standard specification in the literature, common in particular to previous tests for bubbles,[3] and is the fundamental identification restriction maintained in the estimations that follow.

Expectations are assumed to be rational:

$${}_tp_{t+1} - p_t = \mathrm{E}[(p_{t+1} - p_t)|\Omega_t], \tag{3}$$

where Ω_t is the information set at time t.

The 'market fundamentals solution' for the price is given by

$$p_t = \beta + \frac{1}{1-\alpha}\sum_{i=0}^{\infty}\left(\frac{\alpha}{\alpha-1}\right)^i ({}_tm_{t+i} - {}_t\varepsilon_{t+i}), \tag{4}$$

[3]See, for example, Sargent and Wallace (1973), Salemi and Sargent (1979), Flood and Garber (1980a, 1980b, 1983), Flood, Garber, and Scott (1984), Burmeister and Wall (1982, 1987), LaHaye (1985). The high serial correlation of ε_t had first been observed by Cagan.

which implies

$$m_t - p_t = \beta - \frac{\alpha}{\alpha-1}\sum_{i=0}^{\infty}\left(\frac{\alpha}{\alpha-1}\right)^i {}_t\mu_{t+i} + \varepsilon_t, \tag{5}$$

where

$${}_t\mu_{t+i} = {}_tm_{t+i+1} - {}_tm_{t+i}.$$

Note that if α is negative, as expected, $\alpha/(\alpha-1) < 1$ and, assuming that $\lim_{i\to\infty}\sum_{i=0}^{\infty}[\alpha/(\alpha-1)]^i{}_t\mu_{t+i}$ is finite, this solution is stable.

Eq. (4) is not the unique expression for the price level. An arbitrary additional term b_t can be added:

$$p_t = \beta + \frac{1}{1-\alpha}\sum_{i=0}^{\infty}\left(\frac{\alpha}{\alpha-1}\right)^i ({}_tm_{t+i} - {}_t\varepsilon_{t+i}) + b_t, \tag{4$'$}$$

and this will still be a solution, provided

$${}_tb_{t+1} = \left(\frac{\alpha}{\alpha-1}\right)^{-1} b_t.$$

An alternative expression for the money demand is therefore given by

$$m_t - p_t = \beta - \frac{\alpha}{\alpha-1}\sum_{i=0}^{\infty}\left(\frac{\alpha}{\alpha-1}\right)^i {}_t\mu_{t+i} - b_t + \varepsilon_t. \tag{5$'$}$$

The fundamental idea underlying the test employed in this paper comes from exploiting the difference between (5) and (5′). If the latter is the correct specification, neglecting the bubble term in eq. (5) will lead to inconsistent estimates. Since – absent other specification problems – direct Instrumental Variables estimation of eq. (1) will be consistent in either case, a Hausman specification test comparing the estimate of α in (1) and (5) can be interpreted as a test for bubbles.

To implement the test it is necessary to identify the stochastic process followed by the market fundamentals. In accordance with Flood and Garber (1980a) and Burmeister and Wall (1982), the money process is specified as an AR(1) on the first difference of the rate of money growth:[4]

$$\mu_t - \mu_{t-1} = \delta + \theta(\mu_{t-1} - \mu_{t-2}) + e_t. \tag{6}$$

[4] Flood and Garber (1980a) used a Box–Jenkins identification routine on the period December 1918 to August 1923. Since this paper uses the same data, the result is here assumed. The stability of the parameters over different samples was checked with a Chow test, and could not be rejected. Using slightly different data, Evans (1978) identifies the money process as ARIMA (0,2,0), i.e., sets $\delta = 0$ and $\theta = 1$.

Eq. (6) can have two different interpretations. First of all, it can be seen as a structural equation, describing the behavior over time of a money stock which depends only on its own lagged values and is, more specifically, strictly exogenous with respect to prices. This is the interpretation given by Flood and Garber and Burmeister and Wall. The second possibility is to view (6) as an approximation to the univariate representation of any arbitrary, stationary money process, invoking the Wold decomposition theorem. As long as e_t is white noise, this representation is correct and allows a noncommitted stand on the exogeneity of the regressors.[5] The major benefit of this approach is the implied possibility of a feedback rule from inflation to money supply.

Past rates of money supply $\{\mu_{t-1}, \mu_{t-2}, \dots\}$ belong to a limited information set I_t assumed to be strictly contained in Ω_t. If (6) is a structural equation, then

$$\begin{aligned} \mathrm{E}(p_{t+1} - p_t|\Omega_t) &= \frac{1}{1-\alpha} \sum_{i=0}^{\infty} \left(\frac{\alpha}{\alpha-1}\right)^i \mathrm{E}(\mu_{t+i}|\Omega_t) \\ &= \frac{1}{1-\alpha} \sum_{i=0}^{\infty} \left(\frac{\alpha}{\alpha-1}\right)^i \mathrm{E}(\mu_{t+i}|I_t) \\ &= \mu_{t-1} - \frac{\delta(\alpha-1)}{1-\theta} \\ &\quad + \left[\frac{\theta(\alpha-1)}{\alpha-1-\theta\alpha}\right]\left(\mu_{t-1} - \mu_{t-2} - \frac{\delta}{1-\theta}\right). \end{aligned} \tag{7}$$

Eq. (7) can then be substituted in (1) to find the explicit rational expectations solution. In addition, e_t – the disturbance to the money process – will be uncorrelated with all ε_s, for any t,s. If, instead, (6) is to be seen as a univariate representation of an arbitrary process, then we have to allow correlation between e_t and ε_s ($s \leq t+1$). This is equivalent to saying that knowledge of past and present prices can improve the forecast of the future rates of money growth. Therefore,

$$\frac{1}{1-\alpha} \sum_{i=0}^{\infty} \left(\frac{\alpha}{\alpha-1}\right)^i \mathrm{E}(\mu_{t+i}|\Omega_t) = \frac{1}{1-\alpha} \sum_{i=0}^{\infty} \left(\frac{\alpha}{\alpha-1}\right)^i \mathrm{E}(\mu_{t+i}|I_t) + z_t,$$

[5] This is also the interpretation given by West (1985) to the univariate representation for dividents. See the comment in Flood, Hodrick, and Kaplan (1986).

where

$$z_t = \frac{1}{1-\alpha} \sum_{i=0}^{\infty} \left(\frac{\alpha}{\alpha - 1} \right)^i \left[\mathrm{E}(\mu_{t+i} | \Omega_t) - \mathrm{E}(\mu_{t+i} | I_t) \right].$$

Eq. (7) becomes

$$\mathrm{E}(p_{t+1} - p_t | \Omega_t)$$

$$= \mu_{t-1} - \frac{\delta(\alpha - 1)}{1 - \theta} + \left[\frac{\theta(\alpha-1)}{\alpha - 1 - \theta\alpha} \right] \left(\mu_{t-1} - \mu_{t-2} - \frac{\delta}{1-\theta} \right) + z_t. \quad (7')$$

In general, z_t will be serially correlated and will be correlated with e_s $(s \geq t)$.

As is made clear, even though the same equation can be specified in both cases, the statistical relationship between the variables of the model and the stochastic disturbances as well as the correlations amongst the residuals are different in the two scenarios. This implies different estimation techniques and therefore the possibility of distinguishing between the two assumptions.

It is interesting to note how much more pervasive, more difficult to detect, and more interesting the bubble is under the second specification. If money responds to prices, the presence of the bubble affects the market fundamentals which, themselves, become explosive.[6]

An important limitation of this framework is that it does not allow for the probability of a monetary reform. From Flood and Garber (1980a) to more recent works [for example, Hamilton and Whiteman (1985) and Hamilton (1986)], it has been stressed that, were such probability present but not specified in the market fundamentals solution, it could be mistakenly identified as a bubble. However, no set of data has been more thoroughly tested for the expectation of a change in regime than the price and money series of the German hyperinflation [see Flood and Garber (1980b, 1983), and LaHaye (1985)]. All three works reach the conclusion that 'the probability of no reform is essentially unity prior to mid-August 1923' [LaHaye (1985, fn. 18)]. In other words, the last months of the hyperinflation will have to be ignored, but we will proceed with the reasonable confidence that no expected structural break affects the results.

[6]All estimation in this paper is under the hypothesis of no bubble, and it is therefore possible to maintain the assumption of a stationary money process, with its Wold representation. The question of whether eq. (6) is a legitimate representation for money when there is a feedback rule from inflation and a bubble is not straightforward. Under certain specifications the bubble can be strictly stationary, even though it has infinite variance [see Quah (1986)]. In addition, within certain parameters ranges, the money process can be represented as (6) even if it is not stationary.

3. Empirical results: Money exogenous

The two equations of the model are rewritten here for convenience:

$$m_t - p_t = \beta + \alpha({}_t p_{t+1} - p_t) + \varepsilon_t, \tag{1}$$

$$\mu_t - \mu_{t-1} = \delta + \theta(\mu_{t-1} - \mu_{t-2}) + e_t, \tag{6}$$

where ε_t follows a random walk and e_t is serially uncorrelated.

It is also assumed that money is strictly exogenous with respect to prices, and therefore uncorrelated with ε_t at any time and across different times.

In first differences, eq. (1) will be written as

$$(m_t - p_t) - (m_{t-1} - p_{t-1}) = \alpha[(p_{t+1} - p_t) - (p_t - p_{t-1})] + u_t - \alpha w_t + \alpha w_{t-1},$$

where both u_t and the forecast error w_t are white noise,

$$w_t = p_{t+1} - {}_t p_{t+1}.$$

A consistent estimate of α can be obtained using instrumental variables, but due to the moving average disturbance the estimated standard error will not be consistent. To correct for this, the variance of α is estimated using standard nonparametric methods adjusted to allow for a MA(1) error term.[7]

Note that some care has to be devoted to the choice of the instruments: current inflation $(p_t - p_{t-1})$ is correlated with both u_t and w_{t-1}, while the current rate of money growth $(\mu_{t-1} = m_t - m_{t-1})$, even though assumed orthogonal to u_t, is correlated with w_{t-1}. Hence the need to look at longer lags.[8]

The data analyzed are described in the appendix. They are the traditional series studied in bubble tests of the German hyperinflation.

[7]Denoting x_t the vector of instruments and ϕ_t the estimated composite disturbance:

$$\operatorname{avar}(\alpha) = \left(1/T\sum_{t=1}^{T} x_t^2\right)^{-1}\left[\left(\sum_{t=1}^{T}\phi_t^2/T\right)\left(\sum_{t=1}^{T}x_t^2/T\right) + 2\left(\sum_{t=1}^{T-1}(\phi_t\phi_{t-1})/T\right)\left(\sum_{t=1}^{T-1}(x_t x_{t-1})/T\right)\right]\left(1/T\sum_{t=1}^{T}x_t^2\right)^{-1}.$$

Note that we are assuming homoskedasticity.

[8]Recall that

$$w_{t-1} = p_t - {}_{t-1}p_t = \frac{1}{1-\alpha}\sum_{i=0}^{\infty}\left(\frac{\alpha}{\alpha-1}\right)^i ({}_t m_{t+i} - {}_{t-1}m_{t+i}) - u_t.$$

The first term is a complicated function of e_{t-1} and is therefore correlated with $(m_t - m_{t-1})$.

Table 1

Instrumental variables estimation of money demand; May 1920 to June 1923.

α_1	s.e.	ssr
	Instruments: $c, (\mu_{t-2}-\mu_{t-3}), (\mu_{t-3}-\mu_{t-4})$	
-0.745	0.686	3.356
Instruments:	$c, (\mu_{t-2}-\mu_{t-3}), (\mu_{t-3}-\mu_{t-4}), (\pi_{t-2}-\pi_{t-3}), (\pi_{t-3}-\pi_{t-4})$	
0.151	0.222	1.548

The results are reported in table 1.

The estimation with the smaller set of instruments gives rise to a parameter α_1 which has the expected sign, but is not significantly different from zero at the standard confidence levels. If both lagged rates of money growth and lagged inflations are used as instruments, the estimation is, as expected, more efficient, but the point estimate of α is positive. However, since the confidence interval lies on both sides of zero, no conclusion in this sense can be drawn.

What is surprising is not so much the lack of precision in estimating α, a result already found in the literature [for example, in Sargent (1977) and Christiano (1987)], but the substantial discrepancy in results using the two different sets of instruments. Of course this sheds some doubts on the estimate obtained by instrumenting with lagged inflation rates. To check this hypothesis, an overidentifying restrictions test was performed. The calculated value of the test statistic is 2.858, while the critical values are $\chi_4^2(1\%) = 13.276$ and $\chi_4^2(5\%) = 9.487$. The null hypothesis of correct specification cannot, therefore, be rejected.[9]

An alternative test, comparing more directly the two sets of instruments, is a Hausman specification test, where under the null hypothesis of correct specification the estimation with the larger set is relatively efficient (2SLS), but is inconsistent under the alternative. The calculated value of the test statistic is 3.73, while the critical values are $\chi_1^2(1\%) = 6.635$ and $\chi_1^2(5\%) = 3.841$. Again, the hypothesis that lagged inflations are legitimate instruments cannot be rejected (even though the acceptance is only marginal at the 5% significance level).[10]

[9] From a theoretical point of view, the only obvious cases for inconsistency would be the presence of a partial adjustment in the money demand equation or the violation of the random walk assumption for ε_t. The test is a Wald test, where the null hypothesis is that the parameters of the projection of the residuals on the instruments are zero. As in footnote 7, the variance of these estimated parameters is adjusted to take into account the moving average.

[10] The test requires an approximation, since there is no obvious truncation point for the number of lags included as instruments. However, in a context moving as rapidly as a hyperinflation, some parsimony is probably justified. It should also be considered that there are few data points and that the most recent lags have to be excluded because of the correlation with the residuals.

Table 2

Money exogenous; May 1920 to June 1923; restricted Zellner estimation of money demand and money supply.[a]

δ	θ	α_2	D.W.	ssr
0.002	0.148	-0.874	1.83	1.278
(0.011)	(0.169)	(0.205)		0.412

[a] In parentheses the standard errors. The D.W. test and the first sum of squared residuals refer to the money demand equation.

The third and final test concerns the random walk hypothesis since if this were violated lagged inflation rates would be correlated with the errors. The residuals from the IV estimation with the larger instruments set were regressed on themselves lagged twice to avoid the effect of the MA(1) term. The calculated t-statistic, once more corrected for the moving average, is -0.507 and therefore the null hypothesis of no correlation cannot be rejected. Given the results of these tests, we proceed with the understanding that the estimation with the larger set of instruments cannot be neglected.

To obtain an efficient estimate of α, under the hypothesis of no bubble and exogenous money process, we substitute (7) in (1). Taking first differences, the resulting eq. (8) can be jointly estimated with (6):

$$\begin{aligned}(m_t - p_t) - (m_{t-1} - p_{t-1}) &= \alpha\left\{(\mu_{t-1} - \mu_{t-2}) + \frac{\theta(\alpha-1)}{\alpha - 1 - \theta\alpha}\right.\\ &\quad\left.\times\left[(\mu_{t-1} - \mu_{t-2}) - (\mu_{t-2} - \mu_{t-3})\right]\right\} + u_t, \qquad (8)\end{aligned}$$

$$\mu_t - \mu_{t-1} = \delta + \theta(\mu_{t-1} - \mu_{t-2}) + e_t. \qquad (6)$$

Given the assumption on the exogeneity of money, the regressors are orthogonal to the errors and a restricted Zellner estimation is asymptotically equivalent to maximum likelihood. Notice that if money is endogenous and depends on contemporaneous inflation, this estimation method is inconsistent, even in the absence of a bubble.

The results are reported in table 2.

Looking at the money demand equation, α_2 has the expected sign and is significantly different from zero at the standard confidence levels. As anticipated, the standard error is lower than the ones obtained by the instrumental variables estimation. There is no strong evidence of autocorrelation. On the

Table 3

Correlation between the residuals of the money process and the residuals of money demand; May 1920 to June 1923; $e_t = au_t + r_t$; ordinary least squares.

a	s.e.
-0.423	0.06

other hand, the estimation of the money supply equation over this sample is unsatisfactory. This parallels the result obtained by Flood and Garber (1980a). However, a Durbin's h test on the residuals does not detect any evidence of autocorrelation, and therefore of inconsistency.

The joint hypothesis of no bubble and no contemporaneous feedback from inflation to the money supply can be described by a more specific null:

$$\mathrm{H}_0: \quad \operatorname{plim} \alpha_2 = \operatorname{plim} \alpha_1.$$

Under H_0

$$H \equiv \frac{n(\alpha_2 - \alpha_1)^2}{A_n},$$

where A_n converges in probability to $A_0 = \operatorname{avar}(\alpha_1 - \alpha_2) = \operatorname{avar}(\alpha_1) - \operatorname{avar}(\alpha_2)$ (by the Rao–Blackwell theorem). The calculated H is 0.039 (when only lagged moneys are used as instruments) and 150.142 (when both lagged moneys and lagged prices are used), while the critical values are $\chi_1^2(5\%) = 3.841$ and $\chi_1^2(1\%) = 6.635$.

It follows that H_0 cannot be rejected in the first case, while there is very strong evidence of inconsistency in the second. Since the IV estimation with the larger instruments set has passed the previous diagnostic tests, we cannot exclude that either one or both of the hypotheses of predetermined money and no bubble are contradicted by the data from the German hyperinflation.

The latter result is confirmed by another simple test. If money were strictly exogenous, then e_t and u_t would be uncorrelated. A possible test of this hypothesis is therefore to retrieve the errors from the joint estimation and check their correlation. The result is reported in table 3. The coefficient a is significantly different from zero and has the expected sign [$-u_t$ is the residual in $(p_t - p_{t-1})$]. This test, however, is again conditional on the hypothesis of no bubble, and therefore only reinforces the previous conclusion.

Table 4

Money endogenous; May 1920 to June 1923; GMM estimation of money demand and of the money process.[a]

Instruments: $c, (\mu_{t-2} - \mu_{t-3}), (\mu_{t-3} - \mu_{t-4})$.

δ	θ	α	ssr
0.015	0.589	−0.828	1.350
(0.026)	(0.194)	(0.256)	0.443

[a] In parentheses the standard errors. The first sum of squared residuals refers to the money demand equation, the second to the money process.

4. Empirical results: Money endogenous

The results of the previous section indicate that the possibility of an endogenous money supply should be considered explicitly, before drawing any conclusion about the presence (or absence) of a bubble.

If money is allowed to respond to prices, the model becomes

$$(m_t - p_t) - (m_{t-1} - p_{t-1})$$
$$= \alpha\left\{(\mu_{t-1} - \mu_{t-2}) + \frac{\theta(\alpha-1)}{\alpha - 1 - \theta\alpha}\left[(\mu_{t-1} - \mu_{t-2}) - (\mu_{t-2} - \mu_{t-3})\right]\right\}$$
$$+ u_t + \alpha(z_t - z_{t-1}), \tag{8'}$$

$$\mu_t - \mu_{t-1} = \delta + \theta(\mu_{t-1} - \mu_{t-2}) + e_t, \tag{6}$$

where z_t could be serially correlated and conditionally heteroskedastic, and is orthogonal to lagged but not to future e_s. e_t and u_t are white noise, but e_t could be correlated with u_{t+1}, u_t, and lagged u_s. This means that μ_{t-1} in eq. (8′) will have to be instrumented and that it will be necessary to allow for a very general errors structure.

While the instrumental variables estimation does not change, to obtain an efficient estimate of the parameters it is necessary to refer to Generalized Method of Moments estimation [Hansen and Singleton (1982)]. In this context, the problem is simplified by the fact that the model is linear in the variables, and can therefore be seen as a search for the optimal weighting matrix of a 3SNLLS estimator, which will be asymptotically efficient.

Notice that the choice of the instruments is clearly limited by the autocorrelation of the error z_t. In particular, lagged inflation rates would not be correct instruments.

The results are given in table 4.[11]

It is interesting to note that the estimation of θ improves substantially. The estimated α_3 can only be compared to the results of the IV estimation using the same set of instruments. Since the calculated H is 0.017, the hypothesis of no bubble when money is explicitly allowed to respond to prices cannot be rejected.

5. Conclusions

This paper has presented evidence that the data from the German hyperinflation reject the hypothesis of no bubble when this is tested under the assumption of money exogeneity, but cannot reject it when money is allowed to respond to prices.

While this conclusion pleases our intuition, it should be evaluated critically. The result stems mainly from the considerable difference between the instrumental variables estimations with the larger (i.e., including lagged inflation rates) and the smaller instruments set. This clearly implies that more research is needed on the power of the test and on how it is affected by using different lists of instruments. [12]

In addition, the assumption that the shock to money demand follows a random walk is crucial for the interpretation of the results. It will be important to develop in the future a test of this hypothesis, more powerful than the one described in the paper, or to rely on estimation techniques that are robust to its violation.

As a final comment, there is a presumption that much information is lost by limiting our attention to monthly data. Since optimal sampling should reflect the length of the decision period, and the latter is believed to become shorter as inflation increases, there is an argument in favor of using continuous time estimation. This too is left for future research.

[11] In implementing this estimation, two technical problems must be addressed. First, the theoretical number of allowed autocovariances (n) should be infinity. However, it has been proved that a consistent estimate of the optimal weighting matrix W will be obtained by choosing n^* such that $\lim_{T-\infty}(n(T)/T^{1/4}) = 0$. In our case, this leads to n^* between 2 and 3. Second, as is well known, when $n > 1$, W is not positive definite by construction. Having encountered this problem, a simple modification of the W matrix, guaranteed to be positive semidefinite, was computed [Newey and West (1986)].

[12] West (1985) suggests that the asymptotic distribution of the estimators in the presence of a bubble, and therefore the power of the test, might be easier to evaluate when the regressors are explosive. This intuition has been formally confirmed by Durlauf and Hall (1989), implying that if money is treated as exogenous, our results can be correctly interpreted as a bubble test only when lagged inflation rates are included among the instruments. If money is allowed to respond to prices, then, when there is a bubble, money *is* explosive and having lagged rates of money growth as regressors satisfies this criterion. The dimension of the instruments set should then be irrelevant. Unfortunately, the paper by Durlauf and Hall has come to our attention when the present article was already in print. Discussion of the power of this test in finite samples can be found in Mattey and Meese (1986) and in West's comment on their paper.

Appendix: Description of the data

The data analyzed are the ones common to the literature on bubbles tests during the German hyperinflation [in particular they are used by Flood and Garber (1980a), Flood, Garber, and Scott (1984), Burmeister and Wall (1982, 1987), Hamilton and Whiteman (1985)].

The price series are monthly averages of daily rates (collected twice a week) of the index of wholesale prices, as published in a special issue of *Wirtschaft und Statistik*, *Zahlen zur Geldentwertung in Deutschland 1914 bis 1923.*

The money data are the ones published as an appendix to Flood and Garber (1980a). The mid-month total is given by the sum of the mid-month stocks of Reichsbank notes in circulation (obtained by weekly series) and the stocks of other currencies, by far less important, interpolated to middle-of-the-month values from available end-of-the-month reports.

Price controls were not completely lifted until 1920 and, as discussed, the expectations of a monetary reform were probably very high in the last months of 1923. For these reasons, the sample considered in this work goes from May 1920 (in order to include the first months of 1920 when lags are considered) to June 1923. This is the period typically analyzed in the recent literature on the German hyperinflation.

Even though the data have been chosen in order to provide results comparable with previous tests, it must be remarked that the series described are not matching: monthly averages for prices versus mid-month values for money. The daily observations for prices from which the monthly averages were constructed are not available before 1923, and therefore no mid-month values for prices can be obtained. On the other hand, the availability of weekly series for the Reichsbank notes would allow to construct the monthly averages for the money stock. In the data currently studied, the money series are downward biased, intuitively implying a higher likelihood of detecting a bubble. For this reason, it is believed that while the timing should be corrected in future research, our results might be robust to such modification.

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