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Reviewed work(s):
Source: *The Review of Economic Studies*, Vol. 68, No. 1 (Jan., 2001), pp. 83-108
Published by: Oxford University Press
Stable URL: http://www.jstor.org/stable/2695920
Accessed: 28/12/2012 08:51

Review of Economic Studies (2001) **68**, 83–108
0034-6527/01/00050083$02.00

The Role of Market Size in the Formation of Jurisdictions

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First version received May 1996; *final version accepted January* 2000 (*Eds.*)

Administrative and political reorganization is being actively debated even in the mature, stable economies of Western Europe. This paper investigates the possibility that such a reorganization is tied to the integration of economic markets. The paper describes a model where heterogeneous individuals form coalitions for the provision of a public good and shows that the number and composition of these jurisdictions depend on the overall size of the market. The range of economic activities engaged in by jurisdiction members increases when the size of the market increases, and so does the range of their preferences over the public good. The result is a change in the endogenous borders of the jurisdictions, and a reorganization of all coalitions. The optimal number of jurisdictions is unique and increases with market size. In the absence of compensating transfers, however, the decentralized equilibrium need not be optimal and is not unique, although there is no restriction on individuals' ability to coordinate the formation of coalitions. It remains true that a large enough increase in market size will trigger an increase in the number of jurisdictions.

I. INTRODUCTION

The stable societies of Western Europe, long accustomed to free capitalistic markets and democratic political rules, are not experiencing radical upheavals, but seem to be the theatre of a slow, progressive fragmentation of national institutions. Pressures towards increased local control of public revenues and expenditures are noticeable even in countries with the most centralized administrative traditions—France for example—while vocal requests for formal federalist constitutions and threats of secessions are heard in Belgium, Italy, Spain and the United Kingdom. This development is occurring at the same time as economic markets are becoming increasingly integrated: the enactment of the Single Market directives has improved access to other European markets, and international trade flows are everywhere increasing. The purpose of this paper is to investigate the conjecture that these two tendencies may be related. The paper asks what is the role of markets, and in particular of market size, in shaping jurisdictions.

In Italy, for example, the Northern League is a relatively small (and controversial) political party organized around a platform of increased autonomy for the wealthy Northern part of the country. It has played a disproportionate role in shaping the terms of the current political debate. A common view is that the Northern League was formed in reaction to the abuses of the central government and the redistributive transfers imposed from North to South. However, both abuses and transfers have a long history in Post-War Italy. Why the sudden success of the League at the end of the Eighties? An alternative interpretation seems plausible. With unified markets, the Northern firms have found themselves increasingly trading and competing with other European firms. In their economic activities, needs, preferences over rules and institutions, they have progressively approached their foreign competitors and distanced themselves from the less economically developed regions of Southern Italy. Thus they request increased independence from Rome, and increased integration with the rest of Western Europe. The enlargement of the market enlarges the range of possible coalition partners with whom institutions can be designed and shared, and causes the break-up of the old coalitions. This is the idea analysed in this paper.

Although political separatism provides the most striking example, the fragmentation of institutions that is taking place is more subtle and much more common than the overall break-up of a country. The call for increased local autonomy within national units is a call for reform rather than revolution, and its most remarkable feature is its reliance on flexible, international coalitions taking over traditional functions of national governments. In Belgium, for example, the tension between Flanders and Wallonia is keenly felt, and commentators anticipate "that the process of estrangement [...] will continue, and that Flanders will grow through various regional cross-border alliances" (*The Netherlander*, May 24, 1997). Thus not secession, but specialized alliances: "It is perfectly feasible for Flanders to work together with the Netherlands in certain areas and with Wallonia in others" (ibidem). Stimulated by the common market, a beginning of functional federalism seems to be emerging, where different coalitions are formed to provide different public goods. Casella (1996) discussed the increased and dominant role of arbitration in adjudicating private international disputes in Western Europe, even in the presence of high financial costs. Confronted with a larger volume of international deals, businessmen refrain from submitting their cases to national courts and demand the services of international arbitrators, familiar with the usages of their specific commerce. European observers have summarized the phenomenon effectively: "The main consequence [of economic integration] is that the states are losing their unity. They appear as aggregates of specialized jurisdictions that interact directly in the international arena" (Cassese in Galgano, Cassese, Tremonti and Treu (1993, p. 40)). The observed emergence of private, voluntary coalitions is the motivation behind the approach taken in this paper.

The paper builds a simple general equilibrium model where heterogeneous agents form coalitions that choose, finance and share an excludable public good. Individuals are free to join any coalition they wish, or to form their own group. The public good is necessary for the smooth functioning of the market, but because agents differ in their economic activity, they also differ in their preferred choice of public good. A larger coalition implies lower *per capita* taxes, but a larger average distance between the group's choice of the public good and individuals' preferred option. Whether because of specialization or because of a wider range of available products, an increase in market size is modelled as an increase in the number of economic activities—*i.e.* of economic types—present in the market. The paper studies the Strong Nash equilibrium of the coalition–formation game, as function of market size.

The model shows how, in the absence of compensating transfers, the equilibrium may be suboptimal and not unique: either too few or too many coalitions can exist, even when there are no restrictions on coalition formation. The optimal number of coalitions however is unique, and increases with the increased diversity associated with larger markets. Although the conclusion is not surprising, it makes clear the fallacy of associating integrated markets with centralized institutions. The move to a single, large market *per se* does not imply increased homogeneity in preferences over public goods, exactly because the essential purpose of wider markets is to allow more economic diversity, and this diversity is likely to be associated with differing views about the purpose and specifications of public goods.

By studying in detail the endogenous composition of the groups, the model emphasizes how the recombination of coalitions in response to changes in market conditions follows a pattern of secessions and mergers: subgroups detach themselves from their original coalitions, and unite forming a new group. Within the debate on the appropriate administrative structure of the European Union, for example, this observation suggests that in a single market regional and sectoral differences could lead to alliances formed across national borders.

The paper is linked to four different strands of literature. The more direct tie is with recent papers in political economy studying break-ups and formation of countries. Bolton and Roland (1997), for example, study the potential for countries' break-up when secession is associated with a redistributive gain in favour of the majority. In their model there are two well-identified regions, and secession, if it occurs, always follows regional lines. Thus their question is different from the one studied here, where the main focus is on the endogenous and varying composition of the jurisdictions. Alesina and Spolaore (1997) discuss the size and number of nations in a model driven, as in the present paper, by the trade-off between economies of scale in the provision of public goods and diversity. Their work however does not develop the market side of the model, nor does it focus on equilibria that allow free coalition formation.

The tension between the gains from integration and the costs of coordinating heterogenous agents recalls a parallel literature on the endogenous formation of markets. For example, Economides and Siow (1988) analyse an elegant model where individual endowments are stochastic and the variance of market-clearing prices declines with the size of the market. However, costs of "travelling to markets" insure that several smaller markets are optimal. Tamura (1996) studies market integration in the presence of human capital heterogeneity, when agglomeration economies cause productivity to rise with market size, but coordination costs limit the optimal extent of integration. Both the details of the models and the solution concepts differ from the present paper: in particular, the absence of an explicit public good over which disagreement seems very plausible makes the interpretation of the coordination costs less intuitive. Tamura's paper presents a dynamic analysis, but focuses exclusively on the efficient solution and must rely on numerical simulations. Economides and Siow derive analytically both the optimal solution and the Nash equilibrium of the decentralized game, but do not study equilibria robust to coordinated deviations by coalitions.

The endogenous formation of jurisdictions is the subject studied by the theory of clubs. In its modelling assumptions, this paper borrows from David Starrett's treatment of spatial clubs (Starrett (1974) and (1988)), exploiting the spatial representation to capture graphically the heterogeneity of preferences over the public good. Several papers in this literature study club formation in the presence of trade in private goods. However, most of these papers do not analyse the effect of shocks to the market on the clubs (Ellickson (1979), Bewley (1981), Scotchmer and Wooders (1986) and (1987), Wooders (1989) and (1997)). In general, the link between trade and clubs has been underplayed, but there are exceptions (for example, Wilson (1987*a*) and (1987*b*), Benabou (1993)). Among these exceptions, a recent paper by Perroni and Scharf (2001) analyses a question closely related to that studied in the present paper, although the exact modelling choices are different: What is the effect of capital tax competition on the size of jurisdictions? Perroni and Scharf conclude that a more open market for capital results in larger jurisdictions, and interpret the result as a force against local fragmentation. Here we conclude that an increase in market size leads to an increase in the number of clubs, but because the market has become physically larger, it is also true that the minimum optimal size of each club increases at larger market sizes, as in Perroni and Scharf. However we interpret the result as supporting fragmentation: we argue that it is the recombination of jurisdictions accompanying market changes that creates pressure on existing boundaries and will be experienced as a centrifugal force, regardless of the size of the new jurisdiction. Abstracting from market influences, Jéhiel and Scotchmer (2001) study jurisdiction formation with alternative equilibrium concepts: in particular they compare the outcomes supported by free mobility in and out of jurisdictions with equilibria arising when entry into a jurisdiction requires the unanimous consent of the current members. The former equilibrium concept is equivalent to Nash equilibrium, but the latter considers fewer possible deviations than the Strong Nash equilibrium studied in this paper, and the pathological structures that cannot be ruled out under unanimous consent can be excluded in the present analysis. In addition, the focus on trade and market size that is at the heart of the present paper leads to payoffs that are dependent on the entire structure of jurisdictions, and not just on the composition of the specific jurisdiction an individual belongs to, contrary to the model studied by Jéhiel and Scotchmer.

To capture the concept of market size in the easiest possible manner, the paper borrows from Paul Krugman's work on monopolistic competition in trade (Krugman (1980) and (1981), Helpman and Krugman (1986)). With differentiated goods, the measure of available varieties is a natural index of market size, and comparative statics exercises allow us to study quite simply the effect of changes in such a measure on the formation of jurisdictions.

The paper proceeds as follows. Section II describes the model; Section III derives the market equilibrium, for an arbitrary structure of jurisdictions; Section IV presents the central planner solution; Section V studies the Strong Nash equilibria of the coalition-formation game; Section VI discusses the main results, and Section VII concludes.

II. THE MODEL

The economy is represented by a circle of radius r. A continuum of individuals is distributed uniformly along the circle, and each individual owns one unit of the single differentiated good that is traded and consumed in the economy. Each agent's location represents the specific variety of the good he is endowed with.

All individuals have the same preferences summarized in the utility function

$$U=\left(\int_{\Omega} c(i)di\right)^{1/\theta}, \qquad \theta\in(0,1), \tag{1}$$

where $c(i)$ is individual consumption of variety i and Ω is the measure of the set of available varieties (equal to the circumference of the circle). With $\theta<1$, consumers put a premium on diversifying their consumption (Dixit and Stiglitz (1977)).

All exchanges take place in the market, physically located at the centre of the circle, and to arrive there consumers need roads.[1] The cost of a road, C, is linear in the road's length

$$C(r)=\alpha r, \tag{2}$$

where α is a constant, positive parameter.

1. Individuals are not allowed to abstain from trade and must bring their entire (after-tax) endowment to the market.

Any number of traders can share a road (a simple way to capture economies of scale). However, traders must first travel from their initial location to the entry of the road, and as they do so their endowment depreciates: an individual travelling a distance σ along the circumference of the circle is left with $e^{-\delta\sigma}$ units of endowment, where δ is the depreciation rate. Thus traders want to form groups to share the cost of a road, but disagree about its ideal location. This disagreement limits the size of the group (which we will call a "club").

A club has three purposes: it finances the road's construction through taxes levied on the members; it decides jointly the location of the club's road, and it enjoys exclusive use of the road. All members of the club contribute a share t of their initial endowment, and all endowments can be transformed costlessly into resources appropriate for road construction. Thus an individual at distance σ from his club's road arrives at the market holding $(1-t)e^{-\delta\sigma}$ units of endowment, where t will differ across clubs if clubs have different sizes. In addition, members of the club vote (sincerely) on the road's location, and the location that commands a majority against all other alternatives is selected. Each individual is free to choose which club he wants to join, or to set up his own club together with any set of other willing individuals.

The assumption that travel to the road's entry is costly captures graphically the existence of heterogeneous preferences over the public good: one member of the club obtains exactly the public good he needs; all others see their preferences only approximately satisfied, and the discrepancy between the public good provided (the specific location of the road) and these preferences is costly.

The timing of the model is the following: First clubs are formed, then club members are taxed and vote on the road's location, then roads are built, endowments are brought to the market and trade and consumption take place. In the first stage of the game, when clubs are formed, traders are allowed to communicate, all together and in separate groups. The negotiation comes to an end when an agreement is reached such that no coalition of any size wants to deviate from the publicly announced partition into clubs. A perfect foresight Strong Nash equilibrium for this model is a set of prices and consumption levels for all goods and traders, a partition of all consumers into clubs and a specified location for each road such that the market clears, individual utility is maximized, roads' location is decided by majority voting within each club and no set of consumers can achieve higher utility from deviating to a different club or from forming one or several new clubs. The goal is to study how the partition of the economy into clubs changes as the size of the market changes, where the latter is parametrized by the radius r.

Two observations on the assumptions. First, within each club all members are taxed equally, independently of their distance from the road, capturing the observation that tax contributions in a jurisdiction are not affected by individuals' degree of agreement with the policies pursued. Similarly, when clubs are formed there are no transfers within or across clubs to ensure participation by desired individuals, alone or in groups. In both cases, it is possible to object that in the real world funds can and are used to affect political choices. However, particularly in the European examples that motivated the paper, political markets are not transparent and transfers reflect several distinct objectives.[2] It is difficult to think of them as an efficient price mechanism. Missing an analysis of the complexities of these political markets, at least as a first step it seems more realistic to ignore the transfers altogether. The assumption has the added benefit of guaranteeing existence of a Strong Nash equilibrium at all market sizes.

2. Think for example of contributions and transfers in the European Union. It is difficult to see the principal beneficiaries (the Southern European countries and Ireland) as countries who would otherwise opt out of the Union. Redistribution within any jurisdiction often reflects, at least in part, equalitarian purposes.

Second, the identification of market size with the radius r, and thus with variety, captures the view of a larger market as one characterized by a larger number of different possible transactions. It is this view that we typically associate with the expansion of markets. On the other hand, modelling market changes as increases in individual endowments, for given variety, would yield almost identical results.

We begin the solution of the model by deriving equilibrium prices and indirect utility for any arbitrary number and structure of clubs.

III. EQUILIBRIUM PRICES AND INDIRECT UTILITY

Consider the problem faced by consumer s, belonging to club S and located at distance $\sigma(s)$ from the road provided by his club. Club S is of size Ω_s and charges its members taxes t_s, given by

$$t_s = \alpha r/\Omega_s, \tag{3}$$

(in equilibrium, taxes will always be smaller than 1). Consumer s maximizes utility function (1) subject to the budget constraint

$$\int_\Omega p(j)c_s(j)dj = E(\sigma(s), \Omega_S), \tag{4}$$

where $p(j)$ is the price of variety j, $c_s(j)$ is consumption of variety j by individual s and $E(\sigma(s), \Omega_s)$ is the value of the endowment he brings to the market

$$E(\sigma(s), \Omega_S) = p(s)(1-t_s)e^{-\delta\sigma(s)}. \tag{5}$$

His demand for variety j equals

$$c_s(j) = \frac{p(j)^{-1/(1-\theta)}}{\int_\Omega p(j)^{-\theta/(1-\theta)}di} E(\sigma(s), \Omega_S). \tag{6}$$

Equilibrium prices must be such that for each variety total demand by all consumers equals supply, or

$$\int_\Omega c_s(j)ds = (1-t_J)e^{-\delta\sigma(j)}, \tag{7}$$

where variety j belongs to club J and is at distance $\sigma(j)$ from club J's road. It is possible to find an explicit solution for equilibrium prices without imposing any structure on the clubs: in equilibrium $p(j)$ is given by

$$p(j) = (1-t_J)^{-(1-\theta)}e^{\delta(1-\theta)\sigma(j)}, \tag{8}$$

(the Appendix shows that at these prices markets indeed clear)[3]. Equation (8) says that the dispersion in relative prices depends on the different level of taxation in the clubs and on the different distance that goods must travel. Within the same club, this second reason is the only one that will cause relative prices to differ.

As expected, the closer θ is to 0, the more important it is to differentiate consumption and the larger is the increase in price that accompanies the reduced supply of goods

3. Equation (8) gives absolute prices, and as always these are not unique, but can be multiplied by any arbitrary constant. Relative prices, determining allocations, are not affected.

located away from a road or heavily taxed. However, since θ is larger than 0, the higher price never compensates completely for the loss of resources: the value of the endowment that consumer s brings to the market is

$$E(\sigma(s), \Omega_S) = (1 - t_s)^{\theta} e^{-\delta\theta\sigma(s)}. \tag{9}$$

$E(\sigma(s), \Omega_s)$ always falls with increased distance from a road and higher taxes.

Given utility function (1), the indirect utility of consumer s can be written as

$$U(s) = \frac{E(\sigma(s), \Omega_S)}{P}, \tag{10}$$

where the price deflator, corresponding to the minimum expenditure required to achieve one unit of utility, is

$$P = \left[\int_{\Omega} p(j)^{-\theta/(1-\theta)} dj\right]^{-(1-\theta)/\theta}. \tag{11}$$

From equation (8), the price deflator becomes

$$P = \left[\sum_{i=1}^{n} (1 - t_i)^{\theta} \int_{\Omega_i} e^{-\delta\theta\sigma(j)} dj\right]^{-(1-\theta)/\theta} = \left[\sum_{i=1}^{n} \int_{\Omega_i} E(\sigma(j), \Omega_i) dj\right]^{-(1-\theta)/\theta}, \tag{12}$$

where Ω_i is the set of traders (and varieties) belonging to club i and n is the total number of clubs in the economy. Substituting (9) and (12) in (10), we obtain indirect utility of consumer s as function of taxes, of the existing number and size of clubs and of the location of roads

$$U(s) = (1 - t_s)^{\theta} e^{-\delta\theta\sigma(s)} \left[\sum_{i=1}^{n} (1 - t_i)^{\theta} \int_{\Omega_i} e^{-\delta\theta\sigma(j)} dj\right]^{(1-\theta)/\theta}. \tag{13}$$

The term preceding the square brackets in equation (13) captures the effect of taxation and distance from the road on the value of the endowment that consumer s brings to the market. The term in square brackets is the inverse of the price deflator and reflects the distribution of the endowment of consumer s over purchases of all varieties existing in the market. For a given number and structure of clubs, increases in taxes and in the cost of distance from a road (the parameter δ) reduce utility through two different channels: they reduce the value of the endowment that the consumer brings to the market and they increase the price deflator. The closer θ is to 1 (the higher is the elasticity of demand), the larger is the first effect and the smaller the second.

IV. THE CENTRAL PLANNER SOLUTION

Before discussing the equilibrium that arises when groups of individuals voluntarily form coalitions, it is instructive to study the partition into clubs and the location of roads that would be chosen by a central planner.

As usual when individuals are heterogenous, the definition of a social welfare function is arbitrary. We assume that the objective of the central planner is to maximize the sum of individual utilities

$$W = \int_{\Omega} U(s) ds. \tag{14}$$

Given equations (10) and (12), this is equivalent to maximizing total real income or, equivalently, to minimizing the aggregate price deflator, and yields

$$W = P^{-1/(1-\theta)} = \left[\sum_{i=1}^{n} (1-t_i)^{\theta} \int_{\Omega_i} e^{-\delta\theta\sigma(j)} dj\right]^{(1/\theta)}. \tag{15}$$

We can solve the central planner's problem by stages. Suppose first that the size of each club is given. Because the exponential depreciation is a convex function of distance and the distribution of agents is uniform, for any location of roads each club should be formed by a connected segment of types concentrated as much as possible around the corresponding road: any consolidation of a club from disjointed segments into a single connected segment reduces the depreciation suffered by the mass of traders located near the edges. But if each club is a connected segment, then each road should be located at its centre.[4] We can then characterize the optimal relative size of each club, for given total number of clubs. Notice that the welfare function W can be written as

$$W = \left[\sum_{i=1}^{n} \int_{\Omega_i} E(\sigma(j), \Omega_i) dj\right]^{(1/\theta)}, \tag{16}$$

and, if each club is a connected segment with its road at the centre

$$\left(\int_{\Omega_i} E(\sigma(j), \Omega_i) dj\right)^{1/\theta} = \left(1 - \frac{\alpha r}{\Omega_i}\right)^{\theta} \frac{2}{\delta}(1 - e^{-\theta\delta\Omega_i/2}). \tag{17}$$

Over the relevant parameter ranges the right-hand side of (17) is concave in Ω_i. Together with the uniform distribution of types, this implies that in the optimal partition all clubs must have identical size: $\Omega_i = (2\pi r)/n$.

The central planner problem thus reduces to finding, for any set of parameter values, the optimal number of identical clubs, where each club is formed by a connected segment of types and all roads are located in the middle of the corresponding club. The welfare function simplifies to

$$W = \left(1 - \frac{n\alpha}{2\pi}\right)\left[\frac{2n}{\theta\delta}(1 - e^{-\theta\delta\pi r/n})\right]^{(1/\theta)}. \tag{18}$$

Notice that the exact location of the clubs is indeterminate. The central planner chooses the number and the structure of clubs, but is indifferent as to their exact composition: given n, any rotation of the clubs' borders yields the same social welfare.

From equation (18) it is clear that the number of clubs affects social welfare through two channels: a higher n implies a higher number of roads and thus lower average distance from a road; but it implies also smaller clubs and therefore higher taxes. The optimal n results from the trade-off between these two effects.

The characteristics of the optimal choice are not difficult to identify. Proposition 1 addresses our central question, the relationship between the optimal number of clubs and the size of the market, parametrized by the radius r:

Proposition 1. *The optimal number of clubs minimizes the price deflator.* (i) *For all* $r < r^M$, *there exist two numbers* $r^*(n)$ *and* $r^*(n+1)$ (*with* $r^*(n) < r^*(n+1)$) *such that* n *is the*

4. Notice that "consolidating" a club into a single connected segment, for given total size, requires altering the identity of the specific individuals belonging to the club. In the absence of compensating transfers, we cannot exclude that some traders may be hurt; however, total welfare in the economy, as defined by equation (14), must always rise.

optimal number of clubs $\forall r \in [r^*(n), r^*(n+1)]$; (ii) *for* $r \geqq r^M$, *the optimal number of clubs is constant and equal to* n^M. *The threshold value* r^M *solves*

$$W(n^M, r^M) = W((n^M - 1), r^M),$$

where n^M *is the largest integer that satisfies*:

$$(2\pi - n\alpha)n^{1/\theta} \geqq [2\pi - (n-1)\alpha](n-1)^{1/\theta}.$$

The proposition is proved in the Appendix. It has two main implications. First, notice that if n clubs are optimal for all $r \in [r^*(n), r^*(n+1)]$, then $(n-1)$ clubs are optimal for all $r \in [r^*(n-1), r^*(n)]$. With $r^*(n-1) < r^*(n) < r^*(n+1)$, the conclusion is that the optimal number of clubs is unique for all market sizes (with the exception of the discrete set of values $\{r^*(n)\}$, for all $n \leqq n^M$, at which partitions into n and $(n-1)$ clubs yield identical welfare). Second, the proposition establishes that, up to a threshold, the optimal number of clubs is larger the larger is the size of the market.[5] Since distance from a road represents in the model the cost of heterogeneity, we can rephrase this result as follows: As the market expands and becomes more heterogenous, each club also increases in size and in variety. When the heterogeneity within each club becomes too large, the public good is not sufficiently well-tailored to the different needs of the club members, prices reflect this inefficiency, and a new club should appear.

The relation between the parameters α, δ and θ and the market size at which an extra club becomes optimal is predictable. At higher α, the cost of forming a new club is higher, and $r^*(n)$ should increase. At higher δ or lower θ, on the other hand, the cost of distance from a road is higher or has a stronger effect on the price deflator, and $r^*(n)$ should fall. Numerical simulations confirmed these intuitions.

How robust is Proposition 1 to changes in functional forms, and in particular in the technology of road construction? What induces a positive correlation between the size of the market and the optimal number of clubs is the assumption that, for given n, *per capita* taxes rise with r less than the cost of distance from a road, that is less than the cost of heterogeneity. Because the membership of the club rises one-to-one with market size, the total cost of the public good can increase without causing a sensible increase in *per capita* taxes. On the other hand, the cost of heterogeneity is suffered independently by each individual. In other words, the conclusion is driven by the economies of scale inherent in the public nature of the public good, an aspect of the problem more fundamental than a specific choice of functional form. Hence a limited version of Proposition 1 should be consistent with increasing, constant or decreasing returns in the roads' technology, as long as returns are not "too decreasing", i.e. as long as the cost of the public good does not rise too fast with market size. Indeed, if we generalize equation (2) to

$$C(r) = \alpha r^{\beta}, \tag{2'}$$

we find that for all $\beta \in [0, 1)$ the optimal number of clubs always increases with market size; for $\beta \geqq 1$ the result continues to hold if the size of the market is smaller than a threshold whose value depends negatively on β.[6]

5. Although the optimal number of clubs increases with r, it is possible to prove that the minimum size of each club is larger at larger market sizes. For most parameter values, the threshold n^M is much larger than 1 and does not prevent the optimal existence of multiple clubs: for example, when $\alpha = \delta = 0{\cdot}1$ and $\theta = 0{\cdot}5$, $n^M = 42$ and $r^M = 1393{\cdot}53$.

6. The threshold approaches zero as β reaches a critical value that depends on the other parameters of the model. For example, with $\alpha = \delta = 0{\cdot}1$ and $\theta = 0{\cdot}5$, we must have $\beta < 2$. Notice that with decreasing returns the upper limit on taxes implies that the only number of clubs compatible with r approaching infinity is 1. Thus for $\beta > 1$ the optimal number of clubs must increase with market size until the threshold size is reached, remain constant up to a second higher critical value of r, and decline thereafter.

V. EQUILIBRIA WITH FREE COALITION FORMATION

As in the previous section, the first step in characterizing the equilibrium formation of coalitions is to verify general properties that must always be satisfied. Because all Strong Nash equilibria must be robust to individual deviation, we can begin by ignoring coordinated actions and identify the structure of clubs required by all Nash equilibria.

(a) *The structure of clubs in Nash equilibria*

In the most intuitive configuration, each club is formed by a connected segment of individuals and each road is located at the centre of its corresponding club. We show in this section that indeed such a configuration is the only one that can arise in equilibrium. In addition, the symmetry of the model suggests a natural partition of the economy into identical clubs. We find that, although asymmetric configurations can exist, asymmetric equilibria can never be stable: small perturbations of the clubs' borders would unravel the equilibrium.

The logical thread behind these results is simple. Because each individual is infinitesimal, when acting alone the only relevant choice is among the existing clubs. Consider two different individuals. In comparing the various clubs, they face the same choice among taxes, but in general a different distance from the expected location of each club's road. Different preferences can come only from the different location of the two individuals. But because the distribution of individuals along the circle is continuous, preferences over the different clubs must be continuous too: in equilibrium each club must be formed by a connected segment of agents; agents at the border between two clubs must be indifferent between them, while agents closer to the centre of the club will strictly prefer the club to which they belong.

The argument is organized around three lemmas.

Lemma 1. *Each club must be formed by a connected segment of individuals.*

The proof (in the Appendix) exploits the logic described above. In particular, it is possible to show that no partition of the circle into disjoint sets can satisfy the constraints on relative distance that are imposed by individual rationality. Suppose for example that all clubs are of equal size, and thus charge equal taxes. Then no member of a club can be further away from his club's road than from any other. If the clubs are formed by disjoint sets, there is no location of roads for which this condition is satisfied. When clubs are allowed to be of different sizes, a trader may prefer a larger club with lower *per capita* taxes even if its road is not the closest. But the logic remains the same: by comparing the incentives of members of different clubs, we can rule out the possibility that individuals belonging to the same club are drawn from disconnected segments.[7]

This result establishes that, on average, members of a club are more similar to other club members than to individuals belonging to different clubs. Notice that since the partition into intervals is efficient, on this account at least the decentralized equilibrium will not deviate from optimality. Formally, the result is important because it greatly reduces the number of possible equilibria. It also makes it possible to establish the following lemma:

7. As remarked by Greenberg and Weber (1986), this conclusion holds in general in models where individuals' ideal choices of the public good can be ordered on a line, and individuals are allowed to move across jurisdictions.

Lemma 2. *Because in each club the location of the road is decided by majority rule, when multiple clubs exist each road is located exactly at the centre of its corresponding club. When all individuals belong to the same club, the location of the road is indeterminate.*

The formal proof is in the Appendix, but the intuition is straightforward. Consider first the case of multiple clubs. Rewrite indirect utility of consumer s isolating the terms that depend on the location of the road in club S:

$$U(s) = (1 - t_s)^{\theta} e^{-\delta\theta\sigma(s)} \times \left[(1 - t_s)^{\theta} \int_{\Omega_s} e^{-\delta\theta\sigma(j)} dj + \sum_{i \neq S} (1 - t_i)^{\theta} \int_{\Omega_i} e^{-\delta\theta\sigma(j)} dj \right]^{(1-\theta)/\theta}. \tag{13'}$$

A road located as close as possible to consumer s's location minimizes the depreciation his endowment will incur (the first term in (13′), outside the square bracket). But a road located in the middle of the club minimizes the average depreciation suffered by all club members, and thus the prices that consumer s faces when purchasing varieties that also belong to club S (the first term inside the square brackets). As usual, the relative importance of the two effects depends on the parameter θ: for θ sufficiently close to 1, each consumer wants the road at his own location; as θ approaches 0, the ideal point approaches the centre of the club. In general, the preferred location of the road is somewhere between the consumer's own location and the centre of the club. But then, since each club is formed by a connected segment (Lemma 1), the median voter must be the individual at the centre of the club. The Appendix shows that preferences are single-peaked, implying that the location preferred by the median voter cannot be defeated by any other. And since the median voter is at the centre of the club, his ideal road location does not depend on θ and always corresponds to his own position. It follows that although θ determines how diffuse preferences are, for all θ the road will be built in the middle of the club.[8] As noticed by Jéhiel and Scotchmer (2001), when clubs are formed by intervals of individuals, selecting each public good through majority voting implies that its choice is efficient, given the size of the corresponding club.

When all traders belong to the same club, the location of the road has no effect on the aggregate price deflator: average distance from the road is independent of the road location. Each individual wants the road built at his own position, and any proposed location commands exactly 50% of the votes against any other. Since no proposed location can be beaten by any other, in what follows we assume that agreement is somehow reached around a specific location, for example through random choice.

Lemma 3. *Consider a partition of the economy where all clubs are connected segments and all roads are located in the centre of the corresponding club. Call $G(\Omega_i) \equiv U_b(\Omega_i) - U_b(\Omega_j)$ the gain from joining club i to trader b located at the border between clubs i and j. We say that a partition is stable if $\partial G(\Omega_i)/\partial\Omega_i < 0$ for all i and neighbouring j. Then*: (i) *A partition into identical connected clubs is a Nash equilibrium for any $n < 2\pi/\alpha$.* (ii) *A partition into identical connected clubs is stable if and only if $(\delta\pi r)/n > n\alpha/(2\pi - n\alpha)$.* (iii) *All other equilibrium partitions into multiple clubs are unstable.*

8. We have reached this conclusion by considering explicitly the factors determining individual preferences. But notice that since the only difference among the members of a club is their location, once again continuity together with the uniform distribution of types suggest that the median voter must be the individual in the centre.

Part (i) of Lemma 3 is immediate. If all clubs are identical they all charge identical taxes, and with roads located at the centre of each club all individuals at the border between two clubs are equidistant from the two roads. Therefore all border traders are indifferent between the two clubs on their two sides. All individuals strictly inside the clubs' borders are closer to their own road than to any other: they strictly prefer the club to which they belong. No individual deviation is profitable, independently of the number of clubs. Because *per capita* taxes cannot be larger than 1, in this equilibrium, as in any other, n must be smaller than $2\pi/\alpha$.

The proof of parts (ii) and (iii) requires evaluating the sign of $\partial G(\Omega_i)/\partial\Omega_i$ at the different possible equilibria and is left to the Appendix. Notice that the stability condition does not depend on θ. For given δ and α, it says that stability requires a larger market size the higher is the number of clubs. For given n, r must be larger the smaller is the cost of distance from a road δ and the higher is the cost of building the road α.

The stability requirement allows us to concentrate on symmetrical equilibria. If we had a discrete number of agents, instead of a continuum, the possibility of individual deviation would be sufficient to rule out asymmetric partitions.[9] In addition, stability will be of help when studying Strong Nash equilibria. For now, notice the following: the border trader's impact on prices is infinitesimal, and his optimal club size is simply the size that maximizes the value of his endowment. The Appendix establishes that the value of his endowment is single-peaked in club size. If a symmetrical partition is stable, then a club marginally smaller than the *status quo* is preferable to one marginally larger—the optimal club size, for the border trader, must be smaller than $(2\pi r)/n$, and the value of his endowment must decline as the size of the club increases beyond $(2\pi r)/n$. This property will be important below.

Notice that while parts (ii) and (iii) of Lemma 3 depend on the specification of equation (2) and on the exponential depreciation induced by distance from the road, this is not true for Lemmas 1 and 2 and for part (i) of Lemma 3. Given preferences and the general structure of the model, these results will hold for arbitrary functional forms.

By ruling out most club structures, the three lemmas greatly simplify the analysis. In a symmetrical equilibrium, the indirect utility of a consumer at distance s from his club's road is then

$$U(\sigma) = (1-t)e^{-\delta\theta\sigma}\left[\frac{2n}{\theta\delta}(1-e^{-\delta\theta\pi r/n})\right]^{(1-\theta)/\theta}, \tag{19}$$

where $t = n\alpha/2\pi$ and $1 \leqq n < 2\pi/\alpha$.

(b) *Strong Nash equilibria*

For any size of the market, there is a unique number of clubs that would be chosen by a central planner; on the other hand, any feasible symmetrical partition into clubs is a Nash equilibrium. To what extent does coordinated deviation by groups of agents reduce the number of equilibria? Will the outcome approach the central planner's choice?

Consider the following static game in strategic form. Each individual's set of strategies consists of all possible coalitions, *i.e.* of all possible partners with whom to form a club: after consultation, each agent i announces a set Ω_i that includes himself, and all

9. At the added cost, though, of having to evaluate the impact of deviation on the identity of the median voter (see for example Jéhiel and Scotchmer (2001)).

announcements are made simultaneously. All individuals who make identical announcements, and only those who do, unite into a club: a club is formed by all players $\{i, j\}$ such that $\Omega_i = \Omega_j$. Once a club is formed, the location of the road is decided by majority vote and its cost is shared equally by all members. A partition of the circle into n clubs is a Strong Nash equilibrium if no group can coordinate a deviation that benefits all its members. A deviating group can have any size, be drawn from any number of different *status quo* clubs, and plan to either unite into a single club, or coordinate the deviation but form several clubs. Because market prices depend on the entire structure of clubs, individual payoffs are affected not only by the size and composition of the specific club an individual belongs to, but by the whole partition into clubs: this is a game in partition function form. At the same time, individuals can directly influence only their taxes and their minimum distance from a road, and, given truthful majority voting, do so through their choice of club. Hence individual strategies amount only to the announcement of coalition partners.

The game is well-defined: the rule uniting only agents who have made identical announcements is a "coalition structure rule" (in the terminology employed by Burbidge, DePater, Gordon and Sengupta (1997)) and maps any strategy profile into a club structure, and hence into a payoff for each player. It specifies the beliefs on which deviation is evaluated: a subcoalition considering deviation from a larger group expects that all remaining members of the group, as well as all other groups having made identical announcements, will remain together even as the subcoalition secedes.[10] In other words, any deviating group takes the actions of its complement and of all other groups as given. In the general spirit of Nash equilibria refinements, the concept of Strong Nash equilibrium assumes that the deviating group does not anticipate further deviations by the remaining players (Aumann (1959)).[11]

In characterizing the equilibrium, the difficulty is that any deviation is theoretically possible: Having defined a candidate equilibrium, we must identify conditions ruling out deviation by a group of any size, located anywhere along the circle, and acting alone or in coordination with any number of other groups. Fortunately, the primary questions we are interested in are easier to address: Does a Strong Nash equilibrium exist? Is it unique? What is its relation to the central planner equilibrium? The following proposition specifies

10. Notice that the coalition structure rule implies that remaining members of the group unite in a single club that differs from the one they unanimously announce: their announced coalition includes the defecting members.

11. Alternative beliefs are possible and are associated with different equilibrium concepts. For example, the α-core corresponds to pessimistic beliefs: a deviating group must gain even if the remaining players minimize the group's payoff; while in the β-core the group is allowed to consider optimal responses to its complements' actions. For comparison, in a Strong Nash equilibrium a coalition evaluating deviation is allowed to play its best response strategy while constraining its complement to the *status quo* (see the discussion in Ichiishi (1983)). In addition, a deviating coalition is assumed to ignore possible further deviations by members of the coalition itself. In contrast, the concept of Coalition-Proof Nash equilibrium (Berheim, Peleg and Whinston (1988)) requires deviations to be self-enforcing, *i.e.* robust to further deviations by subcoalitions. All these alternative equilibria concepts limit the scope for deviation and thus result in sets of equilibria that are strictly larger than the set of Strong Nash equilibria. As initially suggested by Aumann, Strong Nash equilibria should be applicable both to cooperative and non-cooperative games. In the static game described in this paper the equilibrium can be interpreted as non-cooperative, as it is in several works that study similar strategic-form games of coalition formation (for example Kalai, Postlewaite and Roberts (1979), Burbidge, DePater, Myers and Sengupta (1997), Myers and Sengupta (1997)). However, the ability to coordinate deviation relies on pre-play communication and commitment whose details are left unmodelled, and the distinction between cooperative and non-cooperative games is less sharp than typically acknowledged (see the critique in Perry and Reny (1996)). Here we concentrate on Strong Nash equilibrium as an attractive intuitive benchmark akin to the core, without taking a position in the debate. The relationship between the present coalition game and others studied in the literature (Hart and Kurz (1983), Chatterjee *et al.* (1993), Bloch (1996), Perry and Reny (1996), Ray and Vohra (1997)) is discussed in more detail in Casella (1998).

sufficient conditions that allow us to answer these questions. Because the existence of a Strong Nash equilibrium is of general interest, we present the proof of the proposition in the text. However, readers are not required to read the proof to follow the remainder of the article.

Proposition 2. *If* $\alpha \geqq \pi/n$, *then there exist two numbers,* $\underline{r}(n)$ *and* $\bar{r}(n)$, (*with* $\underline{r}(n) < \bar{r}(n)$) *such that a symmetrical partition into n clubs is a Strong Nash equilibrium for any* $r \in [\underline{r}(n), \bar{r}(n)]$. *Moreover,* $\underline{r}(n) < r^*(n)$ *if the partition is stable at* $r^*(n)$, *and* $\bar{r}(n) > r^*(n+1)$.

Proof. We proceed by stages.

(1) Consider first the temptation to deviate to $(n-k)$ clubs, with $k \in \{1, \ldots, n-1\}$. After deviation there must be one or more clubs of size larger than $(2\pi r)/n$. Because the deviating coalition assumes that non-deviating traders will follow the *status quo* partition, the larger clubs must be formed by the group of traders who deviate. Consider the trader at the border of one of these larger clubs, whose size we call $\mu(2\pi r)$, with $\mu > 1/n$. This trader expects to find himself at distance $\mu\pi r$ from the closest road and must belong to the deviating coalition. Deviation requires:

$$\frac{E(\mu\pi r, \mu(2\pi r))}{P'(n-k, r)} > \frac{E(\sigma, 2\pi r/n))}{P(n, r)} \quad \text{or} \quad \frac{E(\mu\pi r, \mu(2\pi r))}{E(\sigma, 2\pi r/n))} > \frac{P'(n-k, r)}{P(n, r)},$$

where $E(\mu\pi r,\ \mu(2\pi r))$ is the after-tax value of the trader's endowment after deviation, $E(\sigma, 2\pi r/n)$ is the after-tax value of his endowment in the *status quo* (where he is at distance σ from a road), $P'(n-k, r)$ is the price deflator in the possibly asymmetric partition into $(n-k)$ clubs, and $P(n, r)$ is the price deflator in the *status quo*. Because $\sigma \leqq \pi r/n$, $E(\sigma, 2\pi r/n) \geqq E(\pi r/n,\ 2\pi r/n)$. Hence

$$R(n, r, \mu) \equiv \frac{E(\mu\pi r, \mu(2\pi r))}{E(\pi r/n, 2\pi r/n)} \geqq \frac{E(\mu\pi r, \mu(2\pi r))}{E(\sigma, 2\pi r/n}.$$

The price deflator is an inverse transformation of the welfare function; hence we know that at all market sizes $P'(n-k, r) \geqq P(n-k, r)$, where the lack of the prime sign indicates a symmetrical partition. It follows that a necessary condition for deviation is: $R(n, r, \mu) > P(n-k, r)/P(n, r)$. It is easy to verify that for all $\mu > 1/n$, $R(n, r, \mu)$ is strictly decreasing in r and, if the *status quo* partition into n clubs is stable at $r^*(n)$, $R(n, r^*(n), \mu) < 1$ (Recall that stability implies that the after tax endowment of a border trader is strictly decreasing in the size of the club for all club sizes larger than $(2\pi r)/n$). From the proof of Proposition 1, we know that $P(n-k, r)/P(n, r)$ is strictly increasing in r and $P(n-k,\ r^*(n))/P(n, r^*(n)) \geqq 1\ \forall k \in \{1, \ldots, n-1\}$ (with equality if $k = 1$). Given the limits $\lim_{r\to 0} R(n, r, \mu) > 1$ and $\lim_{r\to 0} P(n-k, r)/P(n, r) < 1$, we conclude that, for any value of k and μ, the two functions cross once and only once. Call $\tilde{r}(k, n, \mu)$ the value of r such that $R(n, \tilde{r}, \mu) = P(n-k, \tilde{r})/P(n, \tilde{r})$, for given k and μ. By the previous arguments, deviation requires $r < \tilde{r}(k, n, \mu)$, and if the *status quo* partition into n clubs is stable at $r^*(n)$, $\tilde{r}(k, n, \mu) < r^*(n)\ \forall k \in \{1, \ldots, n-1\}, \forall \mu > 1/n$. If we set $\underline{r}(n) \equiv \max\{\tilde{r}(k, n, \mu)\}$ over all $\mu > 1/n$, $k \in \{1, \ldots, n-1\}$, we conclude that no deviation to $(n-k)$ clubs can occur for any $r \geqq \underline{r}(n)$, where $\underline{r}(n) < r^*(n)$. Notice, for future reference, that since $R(n, r, \mu)$ is strictly decreasing in r and is smaller than 1 at $\tilde{r}(1, n, \mu)$, by the definition of $\underline{r}(n)$ it follows that $R(n, \underline{r}(n), \mu) < 1\ \forall \mu > 1/n$.

(2) Consider now a deviation to an alternative partition with n clubs. If the new partition is symmetrical, it must be agreed upon by the coalition of the whole. But the price deflator is unchanged, and traders located at the entrance of a road in the *status quo* would be strictly worse-off. This deviation is ruled out. An asymmetrical partition could result if the deviating coalition agreed to form at least one club of size larger than $(2\pi r)/n$. But, by the same arguments used in (1), if the partition into n clubs is stable at $r^*(n)$, $R(n, \underline{r}(n), \mu) < 1\ \forall \mu > 1/n$, hence the value of the endowment must fall for traders at the border of this larger club for all $r \geqq \underline{r}(n)$. Because in addition $P'(n, r) > P(n, r)\ \forall r$ (since the new partition would be asymmetrical), the border traders would be strictly worse-off. The deviation is ruled out for all $r \geqq \underline{r}(n)$.

(3) Finally, evaluate a deviation to $(n+k)$ clubs. Consider a group that plans to form a single club after deviation (this need not be the whole of the deviating coalition), and call the size of that club $\mu(2\pi r)$. There are three possible cases:

(a) Suppose first $\mu > 1/n$. A necessary condition for deviation is: $R(n, r, \mu) > P(n+k, r)/P(n, r)$. Recall that for all $\mu > 1/n$, $R(n, r, \mu)$ is strictly decreasing in r, and $R(n, r, \mu) < 1\ \forall r \geqq \underline{r}(n)$, if the *status quo* partition into n clubs is stable at $r^*(n)$. $P(n+k, r)/P(n, r)$ is everywhere declining in r and $P(n+k, r^*(n+1))/P(n, r^*(n+1)) \geqq 1$ (with equality if $k = 1$). For any $\mu > 1/n$ and k, there may exist one, multiple, or no $r'(k, n, \mu)$ such that $R(n, r', \mu) = P(n+k, r')/P(n, r')$, but if any exists, then $r'(k, n, \mu) > r^*(n+1)$. Call $\bar{r}'(n) \equiv \min\{r'(k, n, \mu)\}$ over all $\mu > 1/n$ and all k (if the set is not empty). Then no deviation can occur for any $r \in [\underline{r}(n), \bar{r}'(n)]$, where $\bar{r}'(n) > r^*(n+1)$. If no finite $r'(k, n, \mu)$ exists (as must be the case if $n = n^M$), then no deviation can occur for any $r \geqq \underline{r}(n)$.

(b) Suppose $\mu \in [1/(2n), 1/n]$. We begin by showing $E(\mu\pi r, \mu(2\pi r)) \leqq E(\sigma, 2\pi r/n)$ for at least one trader at the border of the new club: Consider the two traders at the borders of the new club, b_1 and b_2, at distance $\mu\pi r$ from the new road and $2\mu\pi r$ from each other. Order them so that the centre of the new club is indexed by $b_1 + \mu\pi r = b_2 - \mu\pi r$. The interval $[b_1 - \mu\pi r,\ b_2 + \mu\pi r]$ is of length $4\mu\pi r \geqq 2\pi r/n$, the distance between two roads in the *status quo*. Hence at least one *status quo* road lies within the interval. But no point in the interval is at distance larger than $\mu\pi r$ from both b_1 and b_2: there must be at least one border trader for whom distance from the road would not decrease after deviation. Given $\mu \in [1/(2n), 1/n]$, *per capita* taxes cannot be lower. Thus there must be at least one trader at the border of the new club for whom $E(\mu\pi r, \mu(2\pi r)) \leqq E(\sigma, 2\pi r/n)$, with equality possible only when $\mu = 1/n$.

Define.

$$R(n, r, \mu, \sigma) \equiv E(\mu\pi r, 2\mu\pi r)/E(\sigma, 2\pi r/n).$$

For all r,

$$R(n, r, \mu, \sigma) \leqq \left(\frac{2\mu\pi - \alpha}{2\mu\pi - \mu\alpha n}\right)^{\theta}.$$

Deviation requires $R(n, r, \mu, \sigma) > P'(n+k, r)/P(n, r)$, and thus

$$\left(\frac{2\mu\pi - \alpha}{2\mu\pi - \mu\alpha n}\right)^{\theta} > P'(n+k, r)/P(n, r) \geqq P(n+k, r)/P(n, r).$$

Consider first $\mu \in [1/(2n), 1/n)$. Then

$$\left(\frac{2\mu\pi - \alpha}{2\mu\pi - \mu\alpha n}\right)^{\theta} < 1.$$

In addition, we know that $P(n+k, r)/P(n, r)$ is everywhere falling in r and $P(n+k, r^*(n+1))/P(n, r^*(n+1)) \geqq 1$ (with equality if $k=1$). Thus, given μ and k, there exists at most one $r''(n, k, \mu)$ such that

$$\left(\frac{2\mu\pi - \alpha}{2\mu\pi - \mu\alpha n}\right)^{\theta} = P(n+k, r''(n))/P(n, r''(n)),$$

and, if it exists, for all discrete k and $\mu \in [1/(2n), 1/n)$, $r''(n, k, \mu) > r^*(n+1)$. Call $\bar{r}''(n) \equiv \min\{r''(k, n, \mu)\}$ over all $\mu \in [1/(2n), 1/n)$ and all discrete k (if the set is not empty). Then any deviation requires $r > \bar{r}''(n)$, where $\bar{r}''(n) > r^*(n+1)$; no deviation is possible if no $r''(n, k, \mu)$, and hence $\bar{r}''(n)$, exists (as we know to be the case if $n = n^M$).

Now suppose $\mu = 1/n$. Then

$$\left(\frac{2\mu\pi - \alpha}{2\mu\pi - \mu\alpha n}\right)^{\theta} = 1,$$

and any partition into $(n+k)$ clubs must be asymmetrical. Deviation requires $1 > P'(n+k, r)/P(n, r) > P(n+k, r)/P(n, r)\ \forall r$. Call $r'''(n, k)$ the market size such that $P'(n+k, r'''(n))/P(n, r'''(n)) = 1$. If $r'''(n, k)$ exists, then we know $r'''(n, k) > r^*(n+1)\ \forall$ *discrete* k. It follows that any deviation requires $r > \bar{r}'''(n) \equiv \min\{r'''(k, n)\} > r^*(n+1)$; no deviation is possible if no $r'''(n, k)$ exists (as we know to be the case if $n = n^M$).

(c) Finally suppose $\mu < 1/(2n)$. If $\alpha \geqq \pi/n$, $t = \alpha/(2\pi\mu) > 1$, and no club of this size is feasible.

To conclude the proof, if any finite $\bar{r}'(n), \bar{r}''(n)$, or $\bar{r}'''(n)$ exists, call $\bar{r}(n) \equiv \min\{\bar{r}'(n), \bar{r}''(n), \bar{r}'''(n)\}$. Then the previous arguments imply that no deviation is possible for any $r \in [\underline{r}(n), \bar{r}(n)]$, where $\bar{r}(n) > r^*(n+1)$ and $\underline{r}(n) < r^*(n)$, if the *status quo* partition into n clubs is stable at $r^*(n)$. If no finite $\bar{r}'(n)$, $\bar{r}''(n)$, or $\bar{r}'''(n)$ exists, then no deviation is possible for any $r \geqq \underline{r}(n)$, and the proposition is satisfied for any arbitrary $\bar{r}(n) > r^*(n+1)$. ||

Two observations conclude the description of the result. First, if $\alpha \geqq \pi/n$ and $n = n^M$, a symmetrical partition into n clubs is a Strong Nash equilibrium for all $r > \underline{r}(n)$. Second, if the *status quo* is not stable at $r^*(n)$, then no necessary relation holds between $\underline{r}(n)$ and $r^*(n)$, but the arguments used in the proof establish $\underline{r}(n) < r^s(n)$, where $r^s(n) \equiv n^2\alpha/[\delta\pi(2\pi - n\alpha)]$, the minimum market size for which n clubs are stable, as defined in Lemma 3.[12]

Let us be clear about what has been proved: Proposition 2 identifies sufficient conditions that guarantee the existence of a Strong Nash equilibrium and ensure that a symmetrical partition into n clubs must be an equilibrium for a set of market sizes strictly larger than the range for which it is optimal. The proposition does not characterize the whole set of market sizes for which the partition is an equilibrium (and which could possibly not be an interval). However, if its sufficient conditions are satisfied, it allows us to answer the three questions we raised above. First, a Strong Nash equilibrium must exist, a result that is noticeable in itself because of the common problems of existence of

12. Notice that violation of the stability condition is compatible with equilibrium. An unstable partition is one where being the border individual in a club of size $2\pi r/n + \varepsilon$ is preferable to being the border individual in a club of size $2\pi r/n - \varepsilon$; a deviation from the *status quo* into a new partition into $(n-k)$ clubs requires that there exists an ε such that being the border individual in a club of size $2\pi r/n + \varepsilon$ is preferable to being a trader at distance $\pi/n - \varepsilon$ from the road in a club of size $2\pi r/n$. The second condition is always more restrictive. In the numerical exercises, n was always found to be stable at $r^*(n)$ for all $n \geqq 3$, but not always for $n = 2$, depending on parameter values.

Strong Nash equilibria. Second, a partition into n clubs can be an equilibrium both when the market size is too small—r is smaller than $r^*(n)$ and $(n-1)$ clubs would yield higher aggregate welfare—and when the market size is too large—r is larger than $r^*(n+1)$ and $(n+1)$ clubs would be preferable. Thus the equilibrium may be suboptimal, resulting in either too many or too few clubs. Finally, we know from Proposition 1 that at any market size there is a unique number of clubs that would be chosen by a central planner. If n clubs are an equilibrium for a set of market sizes larger than the set for which they are optimal, then there must be market sizes for which the equilibrium is not unique: given r, different numbers of clubs would be robust to deviations by subcoalitions. To see this, notice that if the conditions in Proposition 2 are satisfied, $\underline{r}(n+1) < r^*(n+1) < \bar{r}(n)$ and both n and $(n+1)$ clubs are an equilibrium for all $r \in [\max\{\underline{r}(n), \underline{r}(n+1)\}, \min\{\bar{r}(n), \bar{r}(n+1)\}]$, an interval guaranteed not to be empty.

To understand how sensitive to parameter values these conclusions are, we have run a series of numerical exercises. Figure 1 reports the results for the representative parameter values $\alpha = \delta = 0{\cdot}1$, $\theta = 0{\cdot}5$ and depicts the market sizes for which one, two, three and four clubs were found to be an equilibrium (the thin line). The bold segments are the ranges of market sizes for which each number of clubs is optimal. The curve that appears in the figure for n larger than 2 divides the (r, n) space into a stable (below the curve) and an unstable region (above the curve). Because we have not identified analytically conditions that are both sufficient and necessary for an equilibrium, the figure should be taken with some caution. For each number of clubs, the set of possible deviations was limited as much as possible on theoretical grounds, and each of the remaining deviations was then ruled out numerically. Three features of the results are of interest. First, the set of market sizes for which each number of clubs was found to be an equilibrium is an interval. Second, although the constraint on α in Proposition 2 is violated here, the conclusion of the proposition still applies. In particular, the potential for suboptimality and the multiplicity discussed above are very evident in the figure: for example two, three or four clubs can be an equilibrium at $r = 1$, but only three clubs are optimal at that market size.

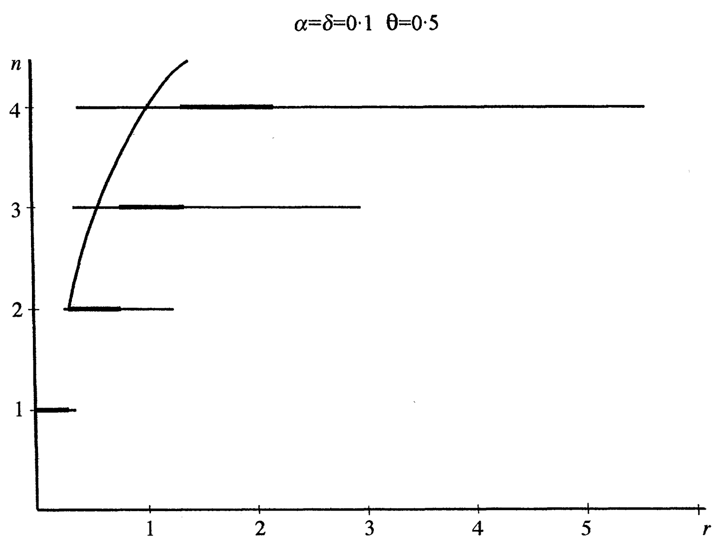

FIGURE 1

Equilibrium number of clubs, strong Nash equilibria and optimal choices

Finally, both the upper and the lower bound of the range of market sizes compatible with equilibrium were found to be increasing in n. Because the equilibrium is not unique, we cannot state that an increase in market size is necessarily associated with an increase in the number of clubs. However, we can make the more limited observation that for the parameter values we have studied the number of clubs supported in equilibrium was always found to increase if the increase in market size was large enough. The positive relationship that we know exists between $r^*(n)$ and n had led us to expect this outcome.[13]

VI. DISCUSSION

The analysis of this model has yielded three main results. The first result is that the equilibrium does not replicate necessarily the central planner's solution, even when individuals are allowed to coordinate their deviations. The bias need not be in a consistent direction: there may be either too few or too many clubs. The conclusion clearly depends on the absence of compensating transfers among individuals either within or across clubs (or equivalently on the absence of a club entrepreneur who organizes the club and sets differential participation fees for the club's members). As mentioned earlier, in the applications we have in mind limiting the extent of redistribution seems preferable to the opposite and equally extreme assumption of fully efficient transfers. In addition, in this model the requirement that all members of a club be treated equally is useful in guaranteeing the existence of equilibria (as for example in Farrell and Scotchmer (1988), or Jéhiel and Scotchmer (2001)). An equilibrium would otherwise exist only when the optimal size of a club is an integer divisor of the whole population, a common problem in club theory (see for example the discussions in Pauly (1970), Scotchmer (1994), Starrett (1988), and Wooders (1978)).[14]

The second main result is that the optimal number of coalitions rises with the size of the market. Given a sufficient extent of heterogeneity, it is not at all obvious that sharing a single larger market necessarily requires unification of institutions, where institutions are identified as groups providing public goods to their members. There is no presumption that a common market should trigger political unification because, even if economies of scale can be realized in centralizing the provision of public goods, the existence of a single unified market does not imply that differences in preferences over public goods have been reduced. In fact, if market expansion is accompanied by economic specialization and by an increase in the range of products and activities that can be found in the market (as for example in Krugman (1980) or (1981), or in Romer (1990)), then we expect larger markets to be matched by more diverse preferences over public goods and by an expansion in the

13. The upper bound in the figure was found to correspond to the market size at which a coalition located symmetrically around a *status quo* border prefers to set-up its own club, where the size of the deviating club, $\mu(\pi r)$, is identified as that size at which deviation occurs at minimum r (in all simulations $\mu < 1/(2n)$ – the constraint on α imposed in Proposition 2 is violated here). The lower bound corresponds to deviation by the coalition of the whole to a symmetrical partition into $(n-1)$ clubs, with the specific location that minimizes the increase in distance for the worst-off individuals. For these market ranges deviations by the coalition of the whole to asymmetrical partitions or to symmetrical partitions with $(n+k)$ clubs were always ruled out; so were enlargements of the *status quo* clubs or deviations to $(n+1)$ clubs by a subgroup not located symmetrically around a *status quo* border; so finally were coordinated deviations by subgroups to $(n+k)$ or $(n-k)$ clubs. A general ranking of deviations is not possible, and it was found to depend on parameter values: for example, for θ high enough the smallest market size compatible with 2 clubs is defined by deviation by a subcoalition enlarging one of the *status quo* clubs (and not by deviation to a single club by the coalition of the whole).

14. In our model, there is a continuum of heterogenous types. The set-up differs from the more usual one where optimal clubs segregate agents according to their types, and where equal treatment is an equilibrium outcome, given profit maximizing club entrepreneurs and a sufficient number of agents of each type. For a discussion of equal treatment in clubs, see Barham and Razzolini (1998).

number of independent coalitions responsible for their provision. The common implicit link between economic and administrative integration is likely to be mistaken.

It is true however that an increase in market size should come together with a reorganization of all coalitions, and this is our third result: not only new clubs are formed, but all clubs must have different borders from the ones that were optimal at smaller market sizes. The branching off and mixing of coalitions is an important feature of this model, one that is quite faithful to the real life developments of groups. It implies that analyses taking as their unit the original coalition artificially constrain the scope for reorganization. Thus, for example, understanding the possible shapes of administrative reorganization in the European Union will require abandoning the traditional focus on national governments, and studying not only regions, but the possibility of coalitions of regions across national borders. Notice also that the reorganization of all groups makes the application of the model to political secessions problematic. Not only the assumption of a single public good, but also the implications of the model suggest the focus on specialized jurisdictions responsible for a well-defined and limited range of public functions.

VII. SUMMARY

This paper has described the equilibrium of a game where heterogeneous individuals form coalitions for the provision of a public good, and where the number and composition of the coalitions they form depend on the overall size of the market. The main insight of the paper is that changes in economic conditions—here represented by the extent of the market—put pressure on existing jurisdictions because they change both the desired public goods and, more importantly, the trade-offs that determine the borders of the jurisdictions. Thus individuals will try to renegotiate the agreements that support the existing jurisdictions and form alternative coalitions. Although the weight of history and long habit will dampen these pressures, their existence can help explain the transformation taking place in Western Europe.

To study this general theme, the paper has presented a simple model of coalition formation. A given partition into clubs is an equilibrium only if no set of individuals, acting alone or in coordination with any other set, can profitably deviate. Using the concept of Strong Nash equilibrium, the paper establishes sufficient conditions guaranteeing that an equilibrium exists, although it is not unique and it need not be optimal. At different market sizes, the equilibrium partition into clubs is modified: when the increase in the size of the market is sufficiently large, subgroups of the original coalitions secede and form new groupings. The new equilibrium, corresponding to a complete reorganization of the original partition, is characterized by a higher number of coalitions.

APPENDIX

1. *Market equilibrium*

We want to verify that there exists an equilibrium where the price of variety j, belonging to club J and located at distance $\sigma(j)$ from its road is given by

$$p(j) = (1 - t_J)^{-(1-\theta)} e^{\delta(1-\theta)\sigma(j)}. \tag{8}$$

The supply of good j reaching the market is $(1 - t_J)e^{-\delta\sigma(j)}$. Substituting (8) and (5) in (6), the demand of variety j by consumer s is

$$c_s(j) = \frac{(1 - t_J)e^{-\delta\sigma(j)}(1 - t_s)^{\theta} e^{-\delta\theta\sigma(s)}}{\sum_{i=1}^{n}\left[(1 - t_i)^{\theta} \int_{\Omega_i} e^{-\delta\theta\sigma(z)} dz\right]}. \tag{A1}$$

Total demand of good j by all consumers is then given by

$$\int_{\Omega} c_s(j)ds = \frac{(1-t_J)e^{-\delta\sigma(j)}\sum_{i=1}^{n}\left[(1-t_i)^{\theta}\int_{\Omega_i} e^{-\delta\theta\sigma(z)}dz\right]}{\sum_{i=1}^{n}\left[(1-t_i)^{\theta}\int_{\Omega_i} e^{-\delta\theta\sigma(z)}dz\right]} \tag{A2}$$

which simplifies to $(1-t_J)e^{-\delta\sigma(j)}$, the supply reaching the market.

2. *The central planner solution*

Proof of Proposition 1. The proof proceeds by stages. We begin by characterizing the optimal number of clubs when the size of the market is very small

$$\lim_{r\to 0}\frac{W(n,r)}{W((n-1),r)} = \left(\frac{2\pi - n\alpha}{2\pi-(n-1)\alpha}\right) < 1, \qquad \forall (n-1) < 2\pi/\alpha. \tag{A3}$$

Equation (A3) holds for all $(n-1)$ such that taxation is feasible, implying that $(n-1)$ clubs are always preferable to n when r approaches 0. When the size of the market is very small, an extra club causes a negligible decline in average distance from a road, and the benefit does not compensate for the higher taxes. Since the minimum possible number of clubs is 1, it follows that $n=1$ is optimal for r sufficiently small.

Similarly, we can identify the optimal number of clubs when the size of the market is very large.

$$\lim_{r\to\infty}\frac{W(n,r)}{W((n-1),r)} = \left(\frac{2\pi - n\alpha}{2\pi-(n-1)\alpha}\right)\left(\frac{n}{n-1}\right)^{1/\theta}. \tag{A4}$$

This limit is larger than 1 for all n smaller than a value n^M, where n^M is defined in Proposition 1. The larger taxes associated with a higher number of clubs must be compensated by the decline in aggregate prices. But if n is above n^M, the cost of the taxes dominate. Since the taxes are higher the higher is α, and the effect on prices is smaller the higher is θ, the threshold n^M depends negatively on α and θ.

In other words, equation (A4) states that as the size of the market becomes very large, n clubs are preferable to $(n-1)$ and $(n-1)$ are preferable to $(n-2)$, etc. for all n smaller than or equal to n^M. The optimal number of clubs is n^M.

Notice that taxes are smaller than 1 for all n smaller or equal to n^M. For all n such that taxes are feasible

$$\operatorname{sign}\frac{\partial(W(n,r)/W((n-1),r)}{\partial r} = \operatorname{sign}\frac{\partial}{\partial r}\left(\frac{1-e^{-\theta\delta\pi r/n}}{1-e^{-\theta\delta\pi r/(n-1)}}\right). \tag{A5}$$

Simple manipulation then shows that the sign must be positive if

$$\frac{\partial(n(e^{\theta\delta\pi r/n}-1))}{\partial n} < 0, \tag{A6}$$

a fact that is easily established. Together with equations (A3) and (A4), this observation implies that for any n smaller than n^M there exists a market size $r^*(n)$ such that $(n-1)$ clubs are preferable to n for all r smaller than $r^*(n)$, and n clubs are preferable to $(n-1)$ for all r larger than $r^*(n)$. But then all is left to show to establish Proposition 1 is that $r^*(n)$ must be strictly smaller than $r^*(n+1)$.

By definition, $r^*(n)$ and $r^*(n+1)$ solve

$$\begin{gathered} W(n, r^*(n))/W((n-1, r^*(n)) = 1, \\ W((n+1), r^*(n+1))/W(n, r^*(n+1)) = 1. \end{gathered} \tag{A7}$$

We can verify that $r^*(n)$ must be smaller than $r^*(n+1)$ by studying the behaviour of the two functions $W(n,r)/W((n-1),r)$ and $W((n+1),r)/W(n,r)$. Given that both functions are everywhere increasing in r, $r^*(n)$ must be smaller than $r^*(n+1)$ if

$$W(n, r)/W((n-1), r) > W((n+1, r)/W(n, r), \qquad \forall r, \tag{A8}$$

or, since W is everywhere strictly positive

$$2 \ln W(n, r) > \ln W(n+1, r) + \ln W((n-1, r). \tag{A9}$$

Condition (A9) is satisfied if $\ln W(n, r)$ is concave in n for all integer values of n. Since $W(n, r)$ is concave in n everywhere, $\ln W(n, r)$ is also concave everywhere. Therefore it must be concave for all integer values of n. We can conclude that $r^*(n+1)$ must be larger than $r^*(n)$. Since the result holds for all n smaller than n^M, Proposition 1 is established. ||

3. *Club structure in Nash equilibria*

Proof of Lemma 1. The analysis is limited to equilibria where the number of clubs is finite and where there is no discrete mass of traders who are exactly indifferent between the club to which they are assigned and an alternative one (a scenario that could only occur if two equal size clubs had roads located at the same point). The outcomes we are ruling out are such that an individual acting alone would be indifferent with respect to deviation, but a positive mass of individuals could coordinate a joint move that would be strictly welfare improving. Because eventually we want to concentrate on Strong Nash equilibria, even if we were to allow such outcomes at this stage, they would be ruled out later.

Consider configurations where at least one club is formed by disconnected segments, and call this club A. The road provided by club A is at location X_A. Then it must be the case that there exists an i, member of club A, whose shortest distance from X_A is larger than the shortest distance between X_A and j, where j is a member of an alternative club, club B, who strictly prefers belonging to B than to A (*i.e.* $\sigma_{iA} = \sigma_{ij} + \sigma_{jA}$, where σ_{iA} is the minimum distance between i and X_A (and similarly for the other subscripts)). This is represented at the top of Figure A1.

Since individual i belongs to club A and individual j prefers club B, and each takes prices as given when evaluating individual deviation, the following two conditions must be satisfied

$$\begin{aligned} (1-t_A)e^{-\delta\sigma_{iA}} &\geqq (1-t_B)e^{-\delta\sigma_{iB}}, \\ (1-t_B)e^{-\delta\sigma_{jB}} &> (1-t_A)e^{-\delta\sigma_{jA}}. \end{aligned} \tag{A10}$$

We show in what follows that there exists no possible location of road B, X_B, such that conditions (A10) can be satisfied simultaneously.

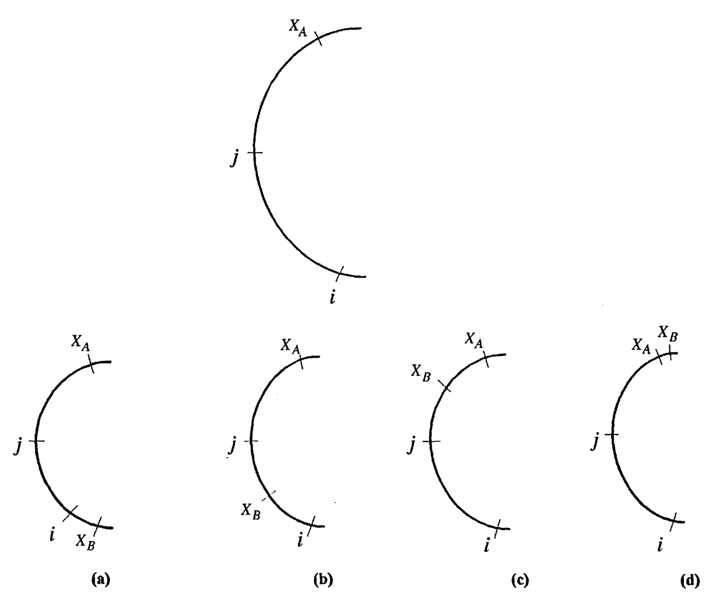

FIGURE A1

Suppose first that X_B is such that $\sigma_{jB} > \sigma_{iB}$ and $\sigma_{jB} > \sigma_{ij}$, case (a) in Figure A1. Then

$$\begin{aligned} \sigma_{jB} &= \sigma_{ij} + \sigma_{iB}, \\ \sigma_{iA} &= \sigma_{ij} + \sigma_{jA}. \end{aligned} \tag{A11}$$

Substituting (A11) in (A10), we derive

$$\begin{aligned} (1-t_A)e^{-\delta(\sigma_{ij}+\sigma_{jA})} &\geqq (1-t_B)e^{-\delta\sigma_{iB}}, \\ (1-t_B)e^{-\delta(\sigma_{ij}+\sigma_{iB})} &> (1-t_A)e^{-\delta\sigma_{jA}}. \end{aligned} \tag{A12}$$

Substituting the second inequality in the first and simplifying, we find that (A12) requires $e^{-\delta\sigma(ij)} > e^{\delta\sigma(ij)}$, which is impossible.

Suppose now that X_B is such that $\sigma_{jB} < \sigma_{ij}$ and $\sigma_{iB} < \sigma_{ij}$, case (b) in Figure A1. Then

$$\sigma_{iA} = \sigma_{iB} + \sigma_{jB} + \sigma_{jA}. \tag{A13}$$

Substituting (A13) in (A10), we derive

$$\begin{aligned} (1-t_A)(e^{-\delta(\sigma_{iB}+\sigma_{jB}+\sigma_{jA})} &\geqq (1-t_B)e^{-\delta\sigma_{iB}}, \\ (1-t_B)e^{-\delta\sigma_{jB}} &> (1-T_A)e^{-\delta\sigma_{jA}}. \end{aligned} \tag{A14}$$

Substituting the second inequality in the first and simplifying, we find that (A14) requires $e^{-\delta\sigma(jB)} > e^{\delta\sigma(jB)}$, which is impossible.

Suppose now that X_B is such that $\sigma_{iB} > \sigma_{ij}$ and $\sigma_{iA} > \sigma_{jA}$, case (c) in Figure A1. Then

$$\begin{aligned} \sigma_{iB} &= \sigma_{ij} + \sigma_{jB}, \\ \sigma_{iA} &= \sigma_{ij} + \sigma_{jA}. \end{aligned} \tag{A15}$$

Substituting (A15) in (A10), we derive

$$\begin{aligned} (1-t_A)e^{-\delta\sigma_{jA}} &\geqq (1-t_B)e^{-\delta\sigma_{jB}}, \\ (1-t_B)e^{-\delta\sigma_{jB}} &> (1-t_A)e^{-\delta\sigma_{jA}}, \end{aligned} \tag{A16}$$

which is impossible.

Finally suppose that X_B is such that $\sigma_{iB} > \sigma_{iA}$ and $\sigma_{jB} > \sigma_{jA}$, case (d) in Figure A1. Then

$$\begin{aligned} \sigma_{iB} &= \sigma_{iA} + \sigma_{AB}, \\ \sigma_{jB} &= \sigma_{jA} + \sigma_{AB}. \end{aligned} \tag{A17}$$

Substituting (A17) in (A10), we derive

$$\begin{aligned} (1-t_A) &\geqq (1-t_B)e^{-\delta\sigma_{AB}}, \\ (1-t_B)e^{-\delta\sigma_{AB}} &> (1-t_A), \end{aligned} \tag{A18}$$

which once again is impossible.

The only other possible location of club B's road is $X_B = X_A$. In this case all members of either club are equally distant from both roads and the two clubs can exist only if they have the same size and thus charge equal taxes to their members. This scenario corresponds to all $i \in A$ and all $j \in B$ being exactly indifferent between either club, a scenario we have excluded at the onset.

We can conclude that no configuration where a club is formed by disconnected segments of individuals can be an equilibrium. Lemma 1 is established. ||

Proof of Lemma 2. Club S is a segment of traders of length Ω_s. For simplicity, parameterize it as extending between 0 and Ω_s and call x the location of the road. The utility of consumer s can then be written as

$$U(s) = (1-t_s)^{\theta} e^{-\delta\theta|s-x|}\left[\sum_{i\neq S}(1-t_i)^{\theta}\int_{\Omega_i} e^{-\delta\theta\sigma(j)}dj + \frac{(1-t_s)^{\theta}}{\delta\theta}(2-e^{-\delta\theta x}-e^{-\delta\theta(\Omega_s-x)})\right]^{(1-\theta)/\theta}, \tag{A19}$$

or

$$U(s) = e^{-\delta\theta|s-x|}a(\theta)[b(\theta)+c(\theta)(2-e^{-\delta\theta x}-e^{-\delta\theta(\Omega_s-x)})]^{(1-\theta)/\theta}, \tag{A20}$$

where $a(\theta)$, $b(\theta)$ and $c(\theta)$ represent the terms in utility not dependent on x. It is simpler to analyse the logarithm of $U(s)$

$$\ln U(s) = -\delta\theta|s-x| + (1-\theta)/\theta \ln[b(\theta)+c(\theta)(2-e^{-\delta\theta x}-e^{-\delta\theta(\Omega_s-x)})] + \ln a(\theta). \tag{A21}$$

Straightforward calculus shows that $\ln U(s)$ is everywhere concave in x and has a unique maximum at x^*, located somewhere between s and $\Omega_s/2$. If s equals $\Omega_s/2$, the unique maximum is at $x = s = \Omega_s/2$; if s is larger than $\Omega_s/2$, $\ln U(s)$ is increasing in x for all x smaller than $\Omega_s/2$ and decreasing in x for all x larger than s; if s is smaller than $\Omega_s/2$, $\ln U(s)$ is increasing in x for all x smaller than s and decreasing in x for x larger than $\Omega_s/2$. Therefore we can conclude that preferences are single-peaked and that the median voter must be the individual at the centre of the club, whose preferred site for the road is always his own location.

The second part of the lemma is immediate. ||

Proof of Lemma 3. (i). Consider trader b at the border between two clubs of sizes Ω_i and Ω_j, where $\Omega_i + \Omega_j = K$ and K is given. In equilibrium he must be indifferent between belonging to either club. With a continuum of individuals, b's club choice has no effect on market prices. Hence $G(\Omega_i) = 0$, or $U_b(\Omega_i) = U_b(\Omega_j)$ if and only if

$$\left(1 - \frac{\alpha r}{\Omega_i}\right)^{\theta} e^{-\delta\theta\Omega_i/2} = \left(1 - \frac{\alpha r}{K - \Omega_i}\right)^{\theta} e^{-\delta\theta(K - \Omega_i)/2}. \tag{A22}$$

The requirement that *per capita* taxes be smaller than 1 implies $\Omega_i \in (\alpha r, K - \alpha r)$, with $K > 2\alpha r$. Condition (A22) is always satisfied at $\Omega_i = \Omega_j = K/2$. If symmetry holds at all border points, $\Omega_i = \Omega_j = (2\pi r)/n$, and the partition is feasible for all n such that $(2\pi r)/n \in (\alpha r, (4\pi r)/n - \alpha r)$, or $n < 2\pi/\alpha$. Because *per capita* taxes are everywhere equal and roads are located in the middle of each club, ruling out deviation by the border trader b is sufficient to rule out deviation by any other trader. Hence a partition into identical connected clubs is a Nash equilibrium for any $n < 2\pi/\alpha$.

(ii). To simplify notation, in what follows we drop the subscript in Ω_i, and, setting $\theta = 1$, define

$$\begin{aligned} f(\Omega) &\equiv \left(1 - \frac{\alpha r}{\Omega}\right) e^{-\delta\Omega/2} \\ g(\Omega) &\equiv f(\Omega) - f(K - \Omega). \end{aligned} \tag{A23}$$

Notice that $G(\Omega) = 0$ if and only if $g(\Omega) = 0$ and $\partial G(\Omega)/\partial\Omega < 0$ if and only if $\partial g(\Omega)/\partial\Omega < 0$.

In a symmetrical equilibrium where $\Omega = K - \Omega$, $\partial g(\Omega)/\partial\Omega = 2\partial f(\Omega)/\partial\Omega$. Hence a symmetrical equilibrium is stable if and only if $\partial f(\Omega)/\partial\Omega < 0$ at $\Omega = (2\pi r)/n$. Since

$$\frac{\partial f(\Omega)}{\partial\Omega} = \frac{e^{-\delta\Omega/2}}{\Omega^2}\left[\alpha r - \frac{\delta\Omega}{2}(\Omega - \alpha r)\right], \tag{A24}$$

the requirement amounts to $(\delta\pi r)/n > n\alpha/(2\pi - n\alpha)$.

(iii). To evaluate the properties of the asymmetric solutions, notice

$$\frac{\partial^2 f(\Omega)}{\partial\Omega^2} = \frac{e^{-\delta\Omega/2}}{\Omega^2}\left[\frac{\delta^2\Omega}{4}(\Omega - \alpha r) - \alpha r\left(\frac{2}{\Omega} + \delta\right)\right]. \tag{A25}$$

Comparing (A24) and (A25), it is easy to see that if $\partial f(\Omega)/\partial\Omega \geqq 0$, then $\partial^2 f(\Omega)/\partial\Omega^2 < 0$. Thus $f(\Omega)$ must have a unique maximum at a value of Ω that we call Ω^*.

In addition, we know

$$\begin{aligned} &\lim_{\Omega\to\alpha r} f(\Omega) = 0, \quad \lim_{\Omega\to(K-\alpha r)} f(\Omega) = \left(\frac{K - 2\alpha r}{K - \alpha r}\right) e^{-\delta(K-\alpha r)/2} > 0, \\ &\lim_{\Omega\to\alpha r} g(\Omega) = -\lim_{\Omega\to(K-\alpha r)} f(\Omega), \quad \lim_{\Omega\to(K-\alpha r)} g(\Omega) = \lim_{\Omega\to(K-\alpha r)} f(\Omega). \end{aligned} \tag{A26}$$

Suppose first $\Omega^* < K/2$ (Figure A2.a). Then $\partial f(\Omega)/\partial\Omega < 0\ \forall\Omega \in (\Omega^*, K - \alpha r]$, and $\partial f(K - \Omega)/\partial\Omega > 0\ \forall\Omega \in [\alpha r, K - \Omega^*)$; hence $\partial g(\Omega)/\partial\Omega < 0\ \forall\Omega \in [\Omega^*, K - \Omega^*]$. The symmetrical equilibrium at $\Omega = K/2$ is stable. In addition, given the limits in (A26), there must be at least two asymmetrical equilibria, on the two sides of $K/2$; as made clear by the figure, the asymmetrical equilibria will be unstable if and only if there are only two of them. Since $g(\Omega) = -g(K - \Omega)$, we can concentrate on $\Omega \in [\alpha r, K/2]$; if we can show that there exists a unique $\hat{\Omega} \in [\alpha r, K/2]$ such that $g(\hat{\Omega}) = 0$, then we have shown that all asymmetrical equilibria are unstable. We have established above that $g(\Omega)$ is strictly decreasing over $[\Omega^*, K/2]$; if $g(\Omega)$ is concave $\forall\Omega \in [\alpha r, \Omega^*)$, then the result follows. Differentiating $g(\Omega)$, we find

$$\frac{\partial^2 g(\Omega)}{\partial\Omega^2} = \frac{e^{-\delta\Omega/2}}{\Omega^2}\left[\frac{\delta^2\Omega}{4}(\Omega - \alpha r) - \alpha r\left(\frac{2}{\Omega} + \delta\right)\right] + \frac{e^{-\delta(K-\Omega)/2}}{(K - \Omega)^2}\left[\alpha r\left(\frac{2}{K - \Omega} + \delta\right) - \frac{\delta^2(K - \Omega)}{4}(K - \Omega - \alpha r)\right]. \tag{A27}$$

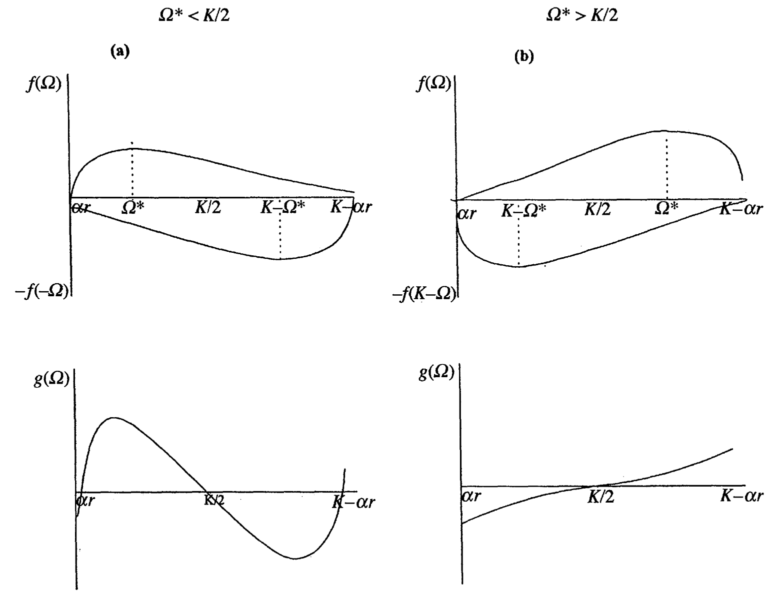

FIGURE A2

The first term equals $\partial^2 f(\Omega)/\partial\Omega^2$ and is negative $\forall\Omega\in[\alpha r, \Omega^*)$. If the second term is negative, then $\partial^2 g(\Omega)/\partial\Omega^2$ is negative, as desired. Suppose instead that it is positive. Then, since $\Omega < K-\Omega$,

$$\frac{\partial^2 g(\Omega)}{\partial\Omega^2} < \frac{e^{-\delta\Omega/2}}{\Omega^2}\left[\frac{\delta^2}{4}[\Omega(\Omega-\alpha r)-(K-\Omega)(K-\Omega-\alpha r)]+2\alpha r\left(\frac{1}{K-\Omega}-\frac{1}{\Omega}\right)\right]. \quad \text{(A28)}$$

But since $\Omega < K-\Omega$, (A28) must be negative, establishing the result. Hence no asymmetrical equilibrium can be stable if $\Omega^* < K/2$.

Suppose now $\Omega^* > K/2$ (Figure A2.b). Then $\partial f(\Omega)/\partial\Omega > 0\ \forall\Omega\in[\alpha r, \Omega^*)$, and $\partial f(K-\Omega)/\partial\Omega < 0\ \forall\Omega\in (K-\Omega^*, K-\alpha r)$; hence $\partial g(\Omega)/\partial\Omega > 0\ \forall\Omega\in[K-\Omega^*, \Omega^*]$. The symmetrical equilibrium at $\Omega = K/2$ is unstable. As made clear by the figure, there is no stable asymmetrical equilibrium if and only if no asymmetrical equilibria exist. Since $g(\Omega)$ must cut the horizontal axis from below, concavity of $g(\Omega)\ \forall\Omega\in[\alpha r, K/2)$ is sufficient to rule out asymmetrical equilibria. With $\Omega^* > K/2$, the first term in (A27) is negative $\forall\Omega\in[\alpha r, K/2)$. Exactly the same argument used above then establishes that $\partial^2 g(\Omega)/\partial\Omega^2$ must be negative $\forall\Omega\in[\alpha r, K/2)$. Hence no asymmetrical equilibria can exist if $\Omega^* > K/2$. Finally, if $\Omega^* = K/2$, $\partial g(\Omega)/\partial\Omega = 0$ at $\Omega = K/2$, but the rest of the argument proceeds unchanged. Part (iii) of the Lemma is established. ||

Notice, for future reference, that because $f(\Omega)$ is single-peaked and stability implies that the unique club size that maximizes the endowment of border trader is smaller than $(2\pi r)/n$, then stability is also sufficient to guarantee that the endowment of the border trader falls monotonically as the club size increases beyond $(2\pi r)/n$.

Acknowledgements. This research was supported by a fellowship from the German Marshall Fund of the United States and was conducted in part at the NBER at Stanford and revised at the Russell Sage Foundation in New York. I am grateful for the generosity of the first institution and the hospitality of the latter two. I thank Roland Benabou, Jan Brueckner, Avinash Dixit, Jurgen von Hagen, Harry Huizinga, David Pines, Tony Venables, Barry Weingast, Myrna Wooders and the participants to numerous seminars and conferences for comments. The editor of this journal and two anonymous referees offered suggestions that have greatly improved the paper. Adriana Kugler and Chuck Rhulin provided able research assistance funded by the Center for German and European Studies at U.C. Berkeley. Cheryl Seleski at Russell Sage drew the figures.

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