Prestige Seeking in College Application and Major Choice

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Abstract

We develop a signaling model of prestige seeking in competitive college applications. A prestigious program attracts high-ability applicants, making its admissions more selective, which in turn further increases its prestige, and so on. This amplifying effect results in a program with negligible quality advantage enjoying a significant prestige in equilibrium. Furthermore, applicants "sacrifice" their fits for programs in pursuit of prestige, which results in the misallocation of program fits. Major choice data from Seoul National University provides evidence for our theoretical predictions when majors are assigned through competitive screening—a common feature of college admissions worldwide.

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1 Introduction

Prestige concerns pervade many high-stakes decisions for individuals. When an individual lands an Ivy League school for college, a top firm for employment, or a top academic journal for research publication, one does not just gain good education, high income, or a wide readership for his/her research. One also gains the *prestige* that comes with a precious brand name. Being selected for the exclusive brand name signals that the individual possesses the desired qualities that are apparently lacking in those who are rejected. Since a prospective employer, spouse, and/or business partners are likely to value these qualities,[1] the brand name itself becomes valuable. Naturally, signaling is more credible the more selective the brand name becomes, so fierce competition ensues in its pursuit, which profoundly shapes one's choices and the social outcome.[2]

The current paper develops a model of prestige seeking and explores its implications for social welfare. While the general thrust of our theory applies to many different contexts, we focus on college applications as our main context. Not only is college application an important arena in which prestige concerns play out (sometimes in high drama), but the stake and implications of their role are particularly significant in this context. Colleges in many countries adopt what we call the *Immediate-Major* (IM) choice system, in which applicants competitively place into majors when they apply for colleges.[3] These choices are crucial not just for their careers at the individual level but also for human resource allocation at the societal level.[4,5]

Although the motive is essentially that of signaling, what is distinctive here and not apparent

[1]Informal and formal evidence for the "elite premium" placed by employers abounds. "Participants overwhelmingly equated university prestige with intelligence. In their eyes, it signaled general cognitive aptitude rather than job-specific skills." (p 87, Rivera, 2016). See Section 4 for formal evidence.

[2]For example, various papers find having a higher rank on the U.S. News rankings impacts students' application decisions and admissions indicators at various types of institutions (Monks and Ehrenberg, 1999; Meredith, 2004; Griffith and Rask, 2007; Bowman and Bastedo, 2009; Luca and Smith, 2013).

[3]Colleges in Chile, China, England, France, Germany, Japan, South Korea, Spain, and Turkey employ this system; students in these countries effectively apply for college-major pairs.

[4]The role played by prestige in major choice is highlighted by Korean surveys. In a survey of college freshmen, Han (2018) finds prestige and fame of a given college/major are as important as individual aptitude and better employment opportunities. Chae (2013) documents that only about 40% of those who graduated from South Korean colleges in 2010 chose majors that aligned with their major fit and aptitude.

[5]While this issue is most pronounced in the IM system, it is also a concern in what we call the *Deferred-Major* (DM) choice system in which students do not choose majors when applying for colleges and "defer" their choices to later years. For example, colleges in the United States, Canada, and Scotland adopt the DM system. According to a report released by ACT Inc, a test-making firm in the US, "almost 80 percent of ACT test-takers who graduated in 2013 said they knew which major they would pursue in college. Of those students, only 36 percent chose a major that fit their interests, according to the study," (see "Study: High School Grads Choosing Wrong College Majors", U.S. News Education.).

in the classical Spencian storyline is the competition agents face in signaling. When students apply, they gravitate toward a prestigious program. The program then becomes more selective in admission, and this increases the signaling value, or prestige, of that program. Consequently, students compete even more fiercely toward that program. Such a feedback loop makes the prestige of a program endogenous and subject to amplification.

Our aim is to capture this feedback loop in a parsimonious way. We consider an Avery and Levin (2010) style model that isolates the basic economics of prestige seeking. In our model, a unit mass of students apply to two college programs,[6] A and B, each with limited capacity. Each student has an iid type—her *score* interpreted as her estimated ability that programs use to screen for their admissions, and her *fits for programs* interpreted as her idiosyncratic preferences for each program.[7] An individual student's score is not publicly disclosed—a standard practice in many contexts—but can be indirectly inferred by outsiders (e.g., potential employers) based on the selectivity of the program admitting that student. This gives rise to a signaling motive in our model. A student's utility of a program consists of her (1) idiosyncratic preference, (2) preference for its vertical quality component, and (3) preference for its prestige—the *average ability of the students enrolling in the program*. For analytical clarity, we assume that the latter two preferences are common to all students. The college application follows a procedure—centralized or decentralized—that produces a *(pairwise) stable matching*—namely a matching that is individually rational and admits no blocking by any program-student pair.[8]

The equilibrium typically features the aforementioned feedback loop. Suppose a program enjoys high prestige. This in turn makes the program more selective in admission, which further increases its prestige, and so on. This "dynamics" means that even when the programs are ex-ante identical, one program emerges as more prestigious between the two in equilibrium, as long as applicants have sufficient prestige concerns. In general, there exists a dominant program that enjoys a higher prestige.

From a social planner's perspective, an important concern is allocative efficiency. Since in our model different individuals value the quality of a program as well as its prestige identically, the allocation of these components has purely distributional consequences with no direct impact on utilitarian welfare. Nevertheless, the competition for quality and prestige has a real consequence on utilitarian welfare. In equilibrium, applicants gravitate toward a program with

[6]The two programs could be two different colleges or two different college majors, depending on the college application system in place, although we will favor the latter interpretation in light of our empirical application.

[7]In the context of major choice, one may interpret fits for programs as one's aptitudes for majors.

[8]While the specific outcome depends on the procedure, many standard procedures (centralized or decentralized) lead to a stable matching.

higher prestige value, and this pursuit of prestige forces them to "sacrifice" their idiosyncratic preferences. Although such prestige-seeking is rational at an individual level, it entails the loss of idiosyncratic preferences and program mismatches at the societal level. In essence, the *zero-sum* nature of the prestige-seeking competition interferes with and harms allocative efficiency. The bigger the prestige concerns are, the fiercer the zero-sum competition gets, and thus the more significant the welfare losses become.[9]

The importance of prestige concerns and their effects are ultimately empirical issues. While these issues may at first glance seem beyond the reach of empirical investigation, we find a unique opportunity to make progress on the empirical front from several admissions channels employed by Seoul National University (SNU).

In particular, we identify two channels, so-called Social Science (SS) and Liberal Studies (LS), through which students choose their social science majors freely after their freshmen year, but for institutional reasons we detail later, SS students are subject to the prestige concerns in the way LS students are not in their major choice. Consistent with our theory, our discrete choice analysis of their choice behavior reveals that the former group exhibits an economically and statistically significant bias in favor of the high-prestige major, Economics, compared with the latter group.

Next, we find evidence that prestige concerns cause students to sacrifice their idiosyncratic preferences—interpreted as 'major fits' in our empirical context. Among those who chose Economics, we find that the SS students subsequently perform poorly in the core major courses *but* not in non-major courses, when compared with their LS counterparts.

Taken together, our findings suggest a significant presence of prestige concerns and their significant impacts on major choice and subsequent performance, at least in the particular context we study.

The rest of the paper is structured as follows. Section 2 develops a theoretical model of prestige seeking and studies its welfare implications. Section 3 provides its empirical evidence in the context of major choice. Section 4 discusses the related literature. Section 5 concludes by discussing further implications of prestige-seeking and possible policy interventions. Appendix A contains proofs of the results in Section 2.

[9]In the context of major choice, the mismatch takes a tangible form. It is natural that a student performs better at a major with a good fit. Hence, a major mismatch is likely to result in poor academic performance in the major courses (measurable in GPA, for instance), a point we return to in our empirical work.

2 Theory of Prestige Concern

2.1 Model

There is a unit mass of students competing for two college programs (or majors) A and B. Each program $j = A, B$ has a mass κ_j of seats. We assume $\kappa_A + \kappa_B \leqslant 1$ so that these two programs are (weakly) overdemanded. There is a lower-valued outside option $\varnothing$, with capacity $\kappa_\varnothing = 1 - \kappa_A - \kappa_B$, interpreted as a less popular program or non-enrollment. Each program $j = A, B$ has an intrinsic **quality** $q_j > 0$ that is common to all students enrolling in that program; this may correspond to the reward associated with the future career of the enrollees, for instance. We assume $q_A \geqslant q_B$, with $\Delta := q_A - q_B$ henceforth referred to as the **quality gap** between the two programs. It is useful to write $q_A = q + \frac{1}{2}\Delta$ and $q_B = q - \frac{1}{2}\Delta$, where $q := \frac{q_A+q_B}{2}$.

Each student has a type $(\varepsilon_A, \varepsilon_B, v) \in T := [0,1]^3$, where v is her score used by both programs for admission, with a higher priority given to a student with a higher score, and ε_j is the student's **fit for program** $j = A, B$ that represents her *idiosyncratic preference* or *aptitude* for the program.[10]

The score v is distributed according to a cdf F which admits a density $f(v) > 0, \forall v \in [0,1]$. Define $\underline{v} \in [0,1]$ such that $1 - F(\underline{v}) = \kappa_A + \kappa_B$, i.e., a score $v > \underline{v}$ is needed for admission to either program. We view the score of each student as a *signal* of her underlying ability (or productivity). To formalize this idea, we let $\theta \in \mathbb{R}$ be a student's ability and assume without loss that $v = \mathbb{E}[\theta|v]$, that is, v is an unbiased estimator of the student's ability. Importantly, we assume that θ is never observable while v is only observable for admission purposes to the college and not observed by outsiders, e.g., the labor market. That applicants' admission scores are not publicly disclosed is realistic in many contexts; it gives rise to the signaling motive in our model.[11][12] The program fits $(\varepsilon_A, \varepsilon_B)$ affect our analysis through their difference $\alpha := \varepsilon_A - \varepsilon_B$. We assume that α is distributed on $[-1,1]$ according to cdf G with density $g(\alpha) > 0, \forall \alpha \in [-1,1]$. We assume that α and v are independently distributed.

An **assignment** is a mapping $\mathfrak{m} : T \to \{A, B, \varnothing\}$ that specifies the program $\mathfrak{m}(t)$ enrolled by each student type $t \in T$. Given an assignment $\mathfrak{m}$, we let T_j denote the set of student types enrolling in program $j = A, B, \varnothing$, i.e., $T_j := \{t \in T : \mathfrak{m}(t) = j\}$. An assignment $\mathfrak{m}$ is feasible if

[10] Our empirical results in Section 3.3.2 will provide some evidence for a possible connection between idiosyncratic preference and aptitude for the program.

[11] The primary justification for nondisclosure is privacy considerations. For instance, the information used for evaluating applicants in the admission process is highly confidential at SNU, the subject of our empirical study. There are some exceptions to this rule. Students' admission scores (or rankings) are publicly available in the Chilean college admissions system (Kaufmann, Messner and Solis, 2013) and at some French *grandes écoles*.

[12] Whether v is observable to students is irrelevant for our analysis.

the measure of T_j for each $j = A, B$ is no greater than κ_j.

For any given assignment $\mathfrak{m}$ and each variable $x = \varepsilon_A, \varepsilon_B, v$, we denote by $\mathbb{E}_j[x] := \mathbb{E}[x|t \in T_j]$ the expectation of x for students enrolling in program $j = A, B, \varnothing$. For instance, $\mathbb{E}_j[\varepsilon_j]$ is the average program fit for students enrolling in program $j = A, B$. Likewise, $\mathbb{E}_j[v]$ is the average score of students enrolling in program $j = A, B, \varnothing$, which, by the unbiasedness assumption, equals the average ability of students enrolling in program j.

Student preferences. Given an assignment $\mathfrak{m}$, the utility of student type $(\varepsilon_A, \varepsilon_B, v)$ from enrolling in program $j = A, B, \varnothing$ is given by:

$$\varepsilon_j + q_j + \tau \left(\mathbb{E}_j[v] - \mathbb{E}[v] \right), \tag{1}$$

where $q_\varnothing = \varepsilon_\varnothing = 0$.[13] The difference $\mathbb{E}_j[v] - \mathbb{E}[v]$ in (1) corresponds to the *prestige*, or *signaling value*, of enrolling in program j, as it measures the average score (or ability) of students in program j over and above the population average. The coefficient $\tau \geqslant 0$ parameterizes the degree to which students are concerned about the prestige of their assigned program.

The functional form in (1) implies that students have a homogeneous preference for quality or prestige *irrespective of* their types. Although this assumption is not without restriction, we make it for analytical clarity, namely to isolate the effect of the misallocation of program fits. The insights derived from our analysis will remain qualitatively valid when we relax this assumption.

Throughout our analysis, we assume that q_A and q_B are sufficiently high that no student type prefers the null program to A or B. We also assume that $1 - G(-\Delta) \geqslant \frac{\kappa_A}{\kappa_A + \kappa_B}$ so that program A cannot accommodate all students with $v \geqslant \underline{v}$ who prefer A over B in terms of their program fits.[14] This assumption will ensure the existence of an equilibrium in which A is more prestigious, i.e., $\mathbb{E}_A[v] \geqslant \mathbb{E}_B[v]$. In case it is violated, there will be an equilibrium in which B is more prestigious.

Welfare criterion. We focus on utilitarian welfare, namely the aggregate utility of all students arising from an assignment:[15]

$$\sum_{j=A,B,\varnothing} \kappa_j \big(\mathbb{E}_j[\varepsilon_j] + q_j + \tau \left(\mathbb{E}_j[v] - \mathbb{E}[v] \right) \big). \tag{2}$$

[13] We assume that $\mathbb{E}_\varnothing[v] = 0$ if $T_\varnothing$ has zero measure.

[14] To see it, note first that absent the signaling motive, a student type would prefer A to B if and only if $\varepsilon_A + q_A > \varepsilon_B + q_B$ or $\alpha > -\Delta$. By definition, the mass of these student types with score $v \geqslant \underline{v}$ is equal to $(\kappa_A + \kappa_B)(1 - G(-\Delta))$ while the capacity of program A is equal to κ_A. The former is (weakly) greater than the latter under the assumption.

[15] Here, we assume that each program $j = A, B$ is fully enrolled, namely, T_j has a measure κ_j for each $j = A, B$. This is the case in all equilibria we will analyze.

Note that the allocation of programs in terms of program quality and prestige does not affect welfare. This is because the supply of quality and prestige is fixed and all students value them equally, so the students effectively play a zero-sum game with regard to these components. The only welfare-relevant component is the allocation of program fit—the first term in (2). Since students differ in their relative fit for alternative programs, how the programs are assigned based on the fit *does* affect welfare.

Equilibrium concept and assignment procedures. The assignment of students across alternative programs depends on the specific matching procedure used. To accommodate a broad class of procedures, both centralized and decentralized, we focus on the following solution concept. We say an assignment $\mathfrak{m}$ is **stable** if there exist cutoff scores $\hat{v}_A$ and $\hat{v}_B$ such that

(i). No blocking: each student is assigned to the program she prefers most among those whose cutoffs are below her score. That is, for each type $t = (\varepsilon_A, \varepsilon_B, v)$, $k = \mathfrak{m}(t)$ implies:

$$k \in \arg\max_{j:\hat{v}_j \leqslant v} \varepsilon_j + q_j + \tau \left(\mathbb{E}_j[v] - \mathbb{E}[v]\right),$$

where $\hat{v}_\varnothing = 0$.

(ii). Market clearing: the measure of T_j equals κ_j for each $j = A, B$.

When the assignment satisfies these two properties, no student has incentives to "block" with an unmatched program $j = A, B$ that is willing to offer a seat to her.[16] Stability is well justified as an equilibrium concept under various institutional settings:

- *Centralized procedure employing student-proposing or program-proposing deferred acceptance (DA) algorithm:* Students submit a list of programs ranked by order of preferences; programs report both their rankings over students and their capacities; and a deferred acceptance algorithm is then used to match the students to the programs. Stability arises under the assumption that students form rational expectations about the prestige of programs that would result from the final assignment and that each program maximizes the aggregate score of enrolled students subject to not wasting its capacity. Under these assumptions, a Nash equilibrium outcome is stable.
- *Decentralized procedure with unrestricted application:* Consider the multi-stage game where in the first stage, all applicants apply to either or both programs; and each program then chooses which students to admit; and in the last stage, students choose from among admitted programs. This procedure is the most common form of decentralized college admissions. Stable

[16] While not explicit, we are thus assuming that each college prefers a high-score student over a low-score student, according to the so-called *responsive* preferences. We also assume that a student with a cutoff score is admitted, which does not conflict with the market clearing assumption due to the atomlessness of F.

matching will arise in any subgame-perfect equilibrium of this game under the same program preferences assumed above since each program will then choose the cutoff that causes its capacity to be exactly filled.[17]

- *Decentralized procedure with restricted application:* Consider the same decentralized procedure as above, *except that students can now apply to only one program.* Restricted applications are common in many decentralized systems, e.g., in Korea, Japan, France, and the US (in the early admissions). Stability will be a valid equilibrium prediction under this regime again under the same program preferences, provided that students observe their scores when applying to a program.[18]

Given the broad applicability of our solution concept, in what follows we shall use **stable matching** and **equilibrium** interchangeably. Throughout, we shall focus on a stable matching in which $\hat{v}_A \geqslant \hat{v}_B$. This is for convenience. When the two programs are sufficiently symmetric, students may coordinate toward an outcome in which either A or B emerges as more prestigious. Such a coordination per se is not of fundamental interest to us, and the analysis of equilibrium with $\hat{v}_A < \hat{v}_B$, if it exists, is analogous.

2.2 Equilibrium and Welfare Analysis

Even though the conditions for stable matchings are standard, our model involves a new feature absent in the standard large matching models, such as Azevedo and Leshno (2016) and Abdulkadiroğlu, Che and Yasuda (2015). That is, a student's preference depends on the enrollment choices of other students, as they affect the prestige of the programs. This feature makes the existence and uniqueness of stable matching nontrivial.[19]

To proceed, we begin by characterizing an equilibrium. In any equilibrium (with $\hat{v}_A \geqslant \hat{v}_B$), it clearly follows that the cutoff $\hat{v}_B$ equals $\underline{v}$. Hence, an equilibrium is characterized by two cutoffs: (1) a *scores cutoff* $\hat{v}_A (\geqslant \hat{v}_B)$ such that program A admits students with score v above $\hat{v}_A$, and (2) a *preference cutoff* $\hat{\alpha}$ such that students with $\alpha \geqslant \hat{\alpha}$ prefer program A to program

[17]Suppose instead a program simply maximizes the total score of enrolled students, without regard to filling its capacity. Then, the program may choose not to fill its capacity by setting a high cutoff in order to boost its prestige and thereby attract students with high scores. This is an interesting possibility—one that can explain the tendency for elite colleges to keep their enrollments small; see Blair and Smetters (2021) for example. To maintain focus, we do not explore this possibility, which is well-justified as long as τ is not too large.

[18]We later discuss a different scenario of restricted application in which students do not observe their scores when they apply to a program. See Section 5.

[19]Strictly speaking, students' prestige concerns can be interpreted as a form of peer preferences. Leshno (2022) establishes the existence of pairwise stable matching in a model with general forms of peer preferences; formally, his existence result applies to our framework. As will be seen, our ultimate aim is comparative statics and welfare analysis regarding prestige seeking, which requires us to take a different approach based on Tarski's fixed point theorem rather than Brouwer's fixed point theorem adopted by Leshno (2022).

B.[20] These two cutoffs identify the types of students T_A enrolling in program A, depicted by the red upper right rectangle in Figure 1.

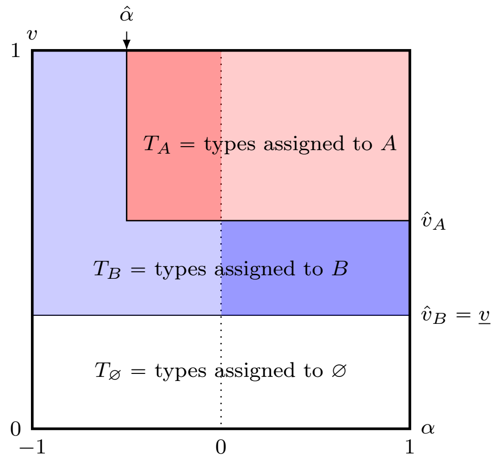

Figure 1: Student Assignment in Equilibrium

These two cutoffs depend on the prestige values of alternative programs since the latter affects students' preferences and thereby their program choice. Since the prestige values are in turn determined by these cutoffs, the equilibrium cutoffs must be characterized by means of a fixed point argument.

Specifically, we focus on the prestige gap between the two programs, denoted by

$$\delta = \mathbb{E}_A[v] - \mathbb{E}_B[v]$$

and study how it is determined as a fixed point of a certain operator $\phi : \delta \mapsto \phi(\delta)$. The operator ϕ, which is defined precisely in Appendix A, is described as follows. One can view the input δ of ϕ as the participants' *belief* about the prestige gap. So, for any arbitrary belief δ, we imagine a "pseudo-equilibrium" in which all students optimally choose their programs given that belief δ and the cutoffs are adjusted to clear the market. The resulting assignment is characterized by two cutoffs $\hat{v}(\delta)$ and $\hat{\alpha}(\delta)$ as described earlier but they depend on the belief δ. One can compute the "new" prestige gap δ' that emerges from this assignment. We then define the operator to output this new prestige gap; namely, we set $\phi(\delta) = \delta'$. Clearly, at a fixed point, the belief is consistent, so it identifies an equilibrium:

Lemma 1. *There exists an equilibrium assignment $\mathfrak{m}$ with prestige gap $\hat{\delta} \geqslant 0$ if and only if $\hat{\delta}$ is a fixed point of ϕ.*

[20] Note that $\hat{\alpha}$ depends on the quality gap and prestige gap. To see this, note that if a student is admitted by both programs, then she will choose A if $\varepsilon_A + q_A + \tau(\mathbb{E}_A[v] - \mathbb{E}[v]) \geqslant \varepsilon_B + q_B + \tau(\mathbb{E}_B[v] - \mathbb{E}[v])$, or $\alpha \geqslant -\Delta - \tau(\mathbb{E}_A[v] - \mathbb{E}_B[v])$.

We next observe that ϕ is monotonic. Intuitively, a higher δ induces more student types to prefer A over B, making A more selective with a higher cutoff $\hat{v}_A$ and thus inducing a higher prestige gap δ'. The existence then follows from Tarski's fixed point theorem.

Theorem 1. *The mapping ϕ is monotonic, so an equilibrium with $\hat{\delta} \geqslant 0$ exists.*

In the standard matching model, a stable matching would be unique if programs have a common ranking of students, just as in our model. However, in our framework, the prestige values of the programs are endogenous, and this feature may give rise to a multiplicity of stable matchings even when one focuses on $\hat{\delta} \geqslant 0$. To see how multiplicity may arise, assume $\Delta = 0$ and g is symmetric around 0. Then, the two programs are completely symmetric. In this case, there always exists a symmetric equilibrium with no prestige gap between the two programs. This equilibrium is depicted by the fixed point P_0 in Figure 2. In such an equilibrium, students with $v \geqslant \underline{v}$ simply choose their programs according to their program fits—namely, $\hat{\alpha} = 0$ in that equilibrium.

However, there may also exist an *asymmetric* equilibrium with a positive prestige gap, particularly if the magnitude of students' prestige concern, τ, is high enough. This equilibrium is depicted by another fixed point P_1 in Figure 2. Intuitively, if all students believe that A has sufficiently higher prestige than B, then even without any quality difference between the two programs, this may make A sufficiently more popular and thus more selective, and this may validate and sustain their initial belief about the prestige gap.

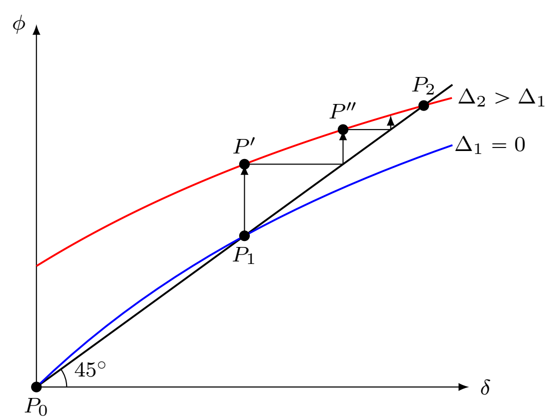

Figure 2: Equilibrium Prestige Gap as a Fixed Point

Proposition 1. *In a symmetric environment where $\Delta = 0$, $\kappa_A = \kappa_B$, and $G(0) = \frac{1}{2}$,*

(i) there exists a symmetric equilibrium in which $\hat{\delta} = 0$;

(ii) if $\tau > \bar{\tau} := \frac{1}{4g(0)(e(\underline{v})-\underline{v})}$, *then there also exists an asymmetric equilibrium in which* $\hat{\delta} > 0$.

In an asymmetric equilibrium, we have $\hat{\delta} > 0 > \hat{\alpha}$ with $\hat{v}_A > \hat{v}_B$. This equilibrium features two types of students who have a mismatch with their majors: students who have a better fit for the program B (i.e., $\alpha < 0$) but end up with A since A offers a higher prestige, illustrated by the darker red area in Figure 1; students who have a better fit for the program A (i.e., $\alpha > 0$) but end up with B since A's cutoff is higher than their scores, illustrated by the darker blue area in Figure 1.

If $\tau > \bar{\tau}$, there exist both symmetric and asymmetric equilibria. In that case, the symmetric equilibrium is *unstable*:[21] that is, if it is perturbed so that the prestige gap becomes a small positive $\epsilon > 0$, then students will adjust their program choices so that the resulting prestige gap becomes higher than ϵ. Intuitively, with students' prestige concern being high enough (i.e., $\tau > \bar{\tau}$), the perturbation will increase the demand for the program A and its selectivity to such an extent that increases the prestige gap even higher than ϵ. By contrast, the asymmetric equilibrium is *stable*; any perturbation away from it will lead to behaviors that restore the original equilibrium. This observation suggests that even when the two programs are ex-ante symmetric, one program is likely to emerge as more prestigious.

Nonetheless, the multiplicity can be avoided under a mild regularity condition as long as $\Delta > 0$:[22]

Proposition 2. *Suppose that* $1 - G(-\Delta) > \frac{\kappa_A}{\kappa_A+\kappa_B}$, $\Delta > 0$, *and* g *is nondecreasing in* $[-1, 0]$. *Then, an equilibrium with* $\hat{\delta} \geqslant 0$ *is unique and satisfies* $\hat{\delta} > 0$.

Note that the sufficient condition for uniqueness may hold even with an arbitrarily small quality gap $\Delta > 0$. For such a $\Delta > 0$, $\phi(0) > 0$, so the unique equilibrium is asymmetric. The prestige gap δ in this equilibrium can be large.[23] In other words, a program with a negligible quality advantage may enjoy a significant prestige advantage in equilibrium.

Comparative statics. In the sequel, we investigate how the equilibrium prestige gap and students' utilitarian welfare change as programs become more stratified (i.e., Δ increases) or students become more concerned about the prestige of their programs (i.e., τ increases). We

[21] Formally, the instability of symmetric equilibrium corresponds to the slope of the mapping ϕ being greater than 1 at $\delta = 0$. Note the stability here should not be confused with the stability introduced earlier as our notion of equilibrium.

[22] Note that this does not rule out an equilibrium with negative prestige gap (i.e., $\mathbb{E}_A[v] - \mathbb{E}_B[v] < 0$).

[23] To illustrate, suppose one perturbs Δ from 0 to a small positive number in Figure 2. Then, the symmetric equilibrium vanishes, and the only surviving equilibrium is in the neighborhood of P_1.

show that such a change indeed causes the equilibrium prestige gap to rise and the utilitarian welfare to fall.

Given the possible multiplicity of equilibria, the comparative statics analysis would require comparing *sets* of equilibria that would result under different parameter values. This requires an order on sets, and we use the *weak-set order* following Che, Kim and Kojima (2021).[24] Say the comparative statics concerns some equilibrium object $x \in \mathbb{R}$. Suppose that the set of possible values for x changes from S to S'. We will say that x becomes *higher* if S' *weak-set dominates* S in the following sense: for any $s \in S$, there is $s' \in S'$ such that $s' \geqslant s$; for any $s' \in S'$, there is $s \in S$ such that $s \leqslant s'$.[25] Analogously, the variable x is said to become *lower* if S weak-set dominates S'. Obviously, this order simplifies to the familiar order if the equilibrium is unique, e.g., if the sufficient conditions of Proposition 2 hold; i.e., $s \leqslant s'$ for $s \in S$ and $s' \in S'$ if S' weak-set dominates S and $|S'| = |S| = 1$.

We are now in a position to state our comparative statics results.

Theorem 2. *Suppose that* (Δ, τ) *increases.*[26] *Then,*

(i) the equilibrium prestige gap becomes higher;

(ii) the equilibrium utilitarian welfare becomes lower, provided that the aggregate quality of programs $\kappa_A q_A + \kappa_B q_B$ *is held constant.*

Theorem 2-(i) states that the prestige gap between A and B increases if either their quality gap Δ increases or simply students' prestige concerns τ increase. It is instructive to understand the mechanism behind this. To illustrate, suppose the quality gap enjoyed by A over B increases from $\Delta_1 = 0$ to $\Delta_2 > \Delta_1$. Since the relative quality of A has increased, some students who previously preferred B now prefer A. This means that $\hat{\alpha}$ falls, and more students now demand A. As a result, A becomes more selective and its cutoff rises. Correspondingly, the scores of students admitted by A increase in absolute and relative terms, increasing the prestige gap. This is illustrated in Figure 2, where an increase in Δ causes the fixed-point map ϕ to shift up, and the prestige gap goes up from P_1 to P'.[27]

However, this is just the direct effect. The endogenous formation of prestige amplifies the

[24] This notion weakens the notion of strong-set order popularized by Milgrom and Shannon (1994). The strong set order, albeit giving a stronger comparison of sets, fails to apply to our comparative statics of equilibria. See Che, Kim and Kojima (2021) for details.

[25] If S and S' are both complete sublattices so that each set contains its supremum and infimum, then this notion is equivalent to both supremum and infimum getting (weakly) higher as S changes to S'. This will be the case in our equilibrium analysis where the set of equilibrium prestige gaps forms a complete sublattice.

[26] That is, both components of the vector increase weakly.

[27] With $\Delta_1 = 0$, there are two equilibria, P_0 and P_1. Here, we focus on the stable equilibrium P_1.

direct effect: the initial increase in the prestige gap induces more students to demand A and makes A even more prestigious, further widening the prestige gap from P' to P''. This process continues until a new fixed point P_2 is reached. In sum, prestige seeking by students amplifies the student response to a given change in the quality gap much beyond the initial direct adjustment. In fact, a similar mechanism is at work with a qualitatively similar outcome, if only students' prestige concern τ increases without there being any changes in the fundamental characteristics of the programs.

Theorem 2-(ii) describes the welfare impact of an increase in (Δ, τ). As noted earlier, a higher prestige gap $\hat{\delta}$ is associated with a higher $\hat{v}_A$ and lower $\hat{\alpha}$. The lower $\hat{\alpha}$ in particular means that students sacrifice their program fits for B to a greater degree to choose A as their program.

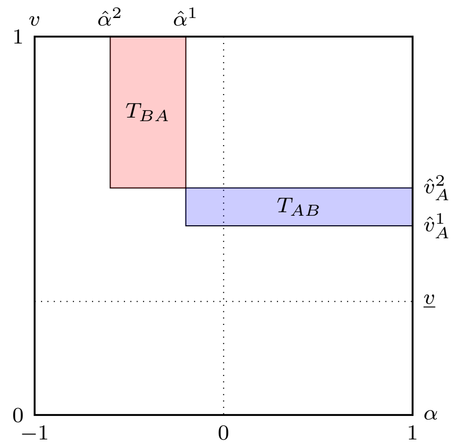

Figure 3: Welfare Implication of Greater Prestige Gap

The welfare effect can be established via a simple *revealed preference argument*. Suppose that parameter values change from (Δ^1, τ^1) to $(\Delta^2, \tau^2) > (\Delta^1, \tau^1)$ and that consequently the equilibrium levels of $(\hat{\alpha}, \hat{v}_A)$ change from $(\hat{\alpha}^1, \hat{v}_A^1)$ to $(\hat{\alpha}^2, \hat{v}_A^2)$, as depicted in Figure 3. Let us focus on the student types who switch their programs across the two equilibria: types T_{AB} switching from A to B, and T_{BA} switching from program B to A. The former types T_{AB} prefer A but *involuntarily* switch to B due to the rise of A's cutoff score. Meanwhile, the latter types T_{BA} switch to A due to the corresponding rise of A's relative signaling value. Consider this switch in the assignment, but imagine hypothetically the prestige values—and therefore students' preferences—have *not* changed. Then, the preference cutoff will still remain at $\hat{\alpha}^1$. Consequently, all these switching types would have preferred their original assignments, so they would have all become worse off. Of course, signaling values have changed, and students have responded to this optimally. Yet, the argument proves that utilitarian welfare falls since the changes in signaling values cancel out due to their zero-sum nature.

An increase in the prestige gap also has a negative impact on students' fit for programs. We show that students' average fit for each program $j = A, B$—that is, $\mathbb{E}_j[\varepsilon_j]$—deteriorates under the parameter value change in Theorem 2 that has caused the students' welfare to fall:

Corollary 1. *Assume that* $\mathbb{E}[\varepsilon_A|\alpha]$ *is increasing in* α *while* $\mathbb{E}[\varepsilon_B|\alpha]$ *is decreasing in* α*. Then, the students' average fit for each program* $j = A, B$ *in equilibria becomes lower as* (Δ, τ) *increases.*

To the extent that one's program fit can affect her academic performance, this effect can be empirically tested, which is precisely what we do in the next section.

3 Evidence of Prestige Seeking: Major Choice in SNU

In this section, we empirically investigate students' major choices using propriety data from SNU in South Korea.[28] Of particular interest are the role and magnitude of signaling in major choice and its impact on students' performances subsequent to their major selection. As theorized earlier, prestige concerns may exist in the Immediate-Major (IM) admissions system in which the students select majors through a competitive screening process as part of their college application. IM admission is a dominant form of major selection for Korean colleges, including SNU. Studying prestige concerns in this context is also relevant, given the widely-held belief that they are an important factor in students' major choices at SNU (and more generally at Korean colleges, see Han, 2018; Chae, 2013). While it is difficult to directly study the IM-based system,[29] we can study the presence and effect of prestige seeking associated with it, using a unique feature of major choice in SNU.

3.1 Institutional Background

SNU is the most prestigious university in South Korea since its establishment in 1946 and has 16 colleges offering 83 undergraduate degree programs.[30] We study student's selection into one of 8 social science majors—Anthropology, Communication, Economics, Geography, Political Science/International Relations, Psychology, Sociology, and Social Welfare—during 2013-2016.[31]

[28] See Online Appendix A.1 for the data description.

[29] Not only is there no comprehensive data on the application through this channel, but the system itself is highly complex. For example, there exist a number of admission types and applicants can apply to seats (the capacity is pre-announced) through early admissions including *General Early* and *Geographic Equality*, *Government-invited Scholarship* and *Transfers*.

[30] SNU's admitted students have top 1% scores in College Scholastic Ability Test (CSAT)—a required examination for students who apply to college—among about 600,000 CSAT takers each year, and the admission acceptance rate is about 15% among those who apply to SNU each year.

[31] A full academic year begins in March and ends in February of the next year in South Korea.

During this sample period, a student could choose a social science major through one of the following 3 channels:

- Immediate-Major (IM) admissions: At the time of college application, a student can directly apply to a particular major/department of SNU. The student is assigned that major if she is admitted based on a competitive screening process corresponding to the major/department. The selectivity of the assignment varies with the major and appears well-understood by outsiders. Indeed, there is a clear perception that the Economics department is most selective among the social science departments.[32] Meanwhile, each individual student's admission score (for example, CSAT score) is not observable to outsiders. This feature makes our theory from Section 2.1 applicable to the major choice made through the IM admissions. Each sample year, about 270 students chose their social science major through this channel.
- Social Science (SS) admissions: A high school student may alternatively apply to the College of Social Sciences (CSS), which houses alternative social science departments, without declaring any major. Each year, about 100 students entered CSS in this manner. When rising to their second year, SS students can choose their department/majors *freely without screening*. Upon their choice of a social science department, they are fully integrated with the students who chose the same department through the IM channel. Their identities from then on are as members of the chosen department, and importantly, completely pooled with, and indistinguishable from, their IM cohort.[33]
- Liberal Studies (LS) admissions: A student may alternatively apply to the College of Liberal Studies (CLS), without declaring any major. Similar to the SS students, an LS student may choose his/her major *freely without screening*, from a large set including social science, humanities, natural science, and engineering, when rising to their second year. In each sample year, about 150 students were admitted in this manner, among which about 60 students chose a social science major. Unlike SS students, however, an LS student is not integrated into a department in official designation but instead maintains a distinct identity as a member of CLS.[34]

[32] No data is available on IM admissions, but the cutoffs estimated by college admissions consulting firms suggest that Economics is most selective among SNU's social science majors, followed by Poli Sci/IR, Psychology, Communication, Sociology, Geography, and Social Welfare, which is in line with the popularity order among SS and LS students.

[33] The official diploma simply indicates the major without revealing the admissions channel the students went through. In addition to its official designation, this admissions channel is sufficiently obscure that not even other fellow SNU students, let alone outsiders, recognize its existence, especially because the SS channel has existed only during 2013-2016 (i.e., our sample period).

[34] In keeping with this identity, his/her diploma would read: "graduate of CLS: Economics major," if he/she chooses Economics in his/her second year. Unlike SS admissions, LS admissions channel is prominently recognized,

In the sequel, we will study the sample of students who selected a social science major through SS and LS channels. Unlike their IM counterparts, these students choose their majors *freely without any screening* in their second year.[35] Hence, at first glance, both SS and LS resemble Deferred-Major (DM) adopted by US colleges. However, SS and LS differ crucially in terms of the signaling motives students face in their major choice. The integration of the SS students with the IM students means that the former students face virtually the same exposure to prestige concerns as the latter, whereas LS students, due to the public knowledge about the lack of screening in their major choice, are relatively immune from prestige concerns, perhaps similarly to the students choosing majors through the DM system, say in US colleges.[36]

For example, if a student chooses an unpopular major, say Sociology, through the SS channel, she risks the outside perception that she likely has "chosen" Sociology because she is below the cutoff for the more popular major, say Economics. However, if she has chosen Sociology through LS, she is not subject to the same degree of stigma. Hence, if prestige concerns are important, one would predict that SS students are more likely to choose Economics over other social science majors than their LS counterparts. One can already see the prediction borne out in Table 1,

Table 1: Major Choice by Admission Channels

	Social Science (SS)		Liberal Studies (LS)	
Sociology	6	(1.78%)	9	(4.25%)
Economics	256	(75.74%)	116	(54.72%)
Poli Sci/IR	43	(12.72%)	33	(15.57%)
Anthropology	1	(0.30%)	4	(1.89%)
Psychology	16	(4.73%)	29	(13.68%)
Geography	2	(0.59%)	2	(0.94%)
Social Welfare	1	(0.30%)	1	(0.47%)
Communcation	13	(3.85%)	18	(8.49%)
Total	338		212	

Note: The sample consists of 550 students who were admitted in AY 2013 to 2016 after the sample selection. The LS students could choose majors outside social sciences; hence, the numbers for LS are conditional on students choosing SS majors.

which presents the percentage shares of different majors chosen by SS and LS students (recall that the majors were chosen "freely" for both SS and LS students.) Note that approximately 76% of SS students chose Economics whereas about 55% of LS students (those who chose social science majors) made the same choice.[37]

and the status of its students as members of the LS is visible both within and outside the campus.

[35] The potential confounding factors that Bordon and Fu (2015) consider—the uncertainty students face in terms of their major fits—are avoided since both SS and LS students have equal timings of major choice.

[36] Online Appendix A.2 illustrates how our theory applies to major choice decisions via SS and LS.

[37] Note that Anthropology, Geography, and Social Welfare were chosen by only 11 students. One may consider

How students were selected into SS and LS is important for interpreting the results we present below. It is commonly believed that the students entering these two programs were comparable in terms of their general aptitude and abilities. However, it is difficult to empirically establish this, due to a lack of data on student characteristics. Table 2 provides descriptive statistics of students by each admission channel. The only "exogenous" student characteristic is the gender ratio, which is statistically similar across SS and LS. Meanwhile, the two groups differ significantly in terms of the first-year courses taken and the ratio between Early versus Regular admissions. However, these differences cannot be attributed to differences in students' unobservable characteristics, since they are largely driven by the differences in the structures and policies of SS and LS.[38]

Given limited evidence of a balancedness of observable characteristics between the two groups, we control for all characteristics of which differences are statistically significant in our following analysis. However, controlling for observable characteristics may not sufficiently address selection into different admission channels, and hence one must explicitly consider the possible endogenous selection into the two groups and their implications for our subsequent results. We discuss several selection hypotheses in Section 3.4 in detail.

Table 2: Student Summary Statistics by Admission Channel

	Liberal Studies, n=212		Social Science, n=338		LS−SS	
	Mean	SD	Mean	SD	Diff.	SE
Female (%)	47.17		43.49		3.68	4.36
Regular (%)	4.72		57.99		-53.27	3.59
Freshman Information						
GPA	3.47	0.45	3.37	0.43	0.11	0.04
# Econ. Courses	0.89	0.99	1.44	0.94	-0.55	0.08
# Math. Courses	1.78	1.17	0.43	0.77	1.35	0.08
# Lib. Art. Courses	6.63	1.65	7.47	1.55	-0.85	0.14
# Business. Courses	0.08	0.31	0.02	0.14	0.06	0.02

Note: The sample consists of 550 students who were admitted in AY 2013 to 2016 after the sample selection described in Online Appendix A.1. 'Freshman GPA' has scales from 0 (F) to 4.3 (A+). 'Regular' is the fraction of students who were admitted through regular admissions; omitted admission methods are early admissions and other admission methods. The last two columns 'LS−SS' presents the results of group mean comparison t-tests. See Table A.1 for comparison with IM students.

grouping those 'small' majors in the analyses that follow. However, our ultimate interest is in comparing SS and LS students who chose Economics as their major, and grouping those small majors does not produce qualitatively different results. Hence, our preferred specification does not group those majors.

[38]Except for 2013 and 2014, CSS exclusively used early admissions to admit students via IM admissions and primarily used regular and other special admissions to admit students via SS admissions. On the other hand, except for 2013, CLS admitted students exclusively via early admissions. In terms of first-year courses taken, SS students were required to take three social science introductory courses (e.g., Principles of Economics), and LS students were required to take CLS seminar courses designed to explore the fields of liberal arts and sciences, and more importantly, one math course (e.g., Mathematics: The Basics and Applications).

3.2 Major Choice

In the following analyses, we consider student i's major choice problem. Denote the (social science) majors by $j = 1, 2, \cdots, J$ and student i's actual major choice by $j(i)$. We do not observe the selection process into LS and SS channels; as mentioned in Footnote 29, they are governed by the complex application and screening process. For our purpose, we will assume that each chosen major choice channel is an exogenous student characteristic. As mentioned, we will address possible scenarios of selection into SS and LS channels in Section 3.4.

Given the free and voluntary nature of their major choice, we apply the discrete choice model to analyze LS and SS students' major choice decisions. Consider the following random-utility model describing student i's utility from enrolling in major j:

$$U_{ij} = \gamma_j + \theta_j SS_i + \sum_l \delta^l x_j^l z_i^l + \varepsilon_{ij}, \tag{3}$$

where γ_j is the major fixed effect; SS_i is a dummy variable which equals 1 if i is an SS student and 0 otherwise; θ_j is the additional fixed effect associated with the SS channel, and we normalize by setting $\theta_{Sociology} = \gamma_{Sociology} = 0$. The variable x_j includes major observable characteristics (the average GPA of the students in major j in the previous year), z_i is a vector of student observable characteristics (admission methods, freshman GPA, and the courses taken in their first year), and ε_{ij} captures i's fit for j. As mentioned earlier, we interpret this term to include a student's pure idiosyncratic taste component as well as her utility from aptitude in a given major, as will be justified later. As is standard, we assume ε_{ij} to be distributed as i.i.d Extreme Value Type-I (EVT1).

The parameter γ_j captures the average common valuation to LS and SS students of major j. Since these students choose their majors freely, we interpret the estimate of γ_j as reflecting an (average) student's intrinsic preferences for major j based on her perception of its general appeals, quality, and employment prospect. Most importantly, the parameter θ_j captures the additional average valuation that SS students assign to major j in addition to γ_j. We interpret θ_j as the signaling value of major j, more precisely the signaling value of that major derived from the IM channel. A significant positive estimate of θ_{econ} would therefore be consistent with the presence of signaling motive in the major choice. Recall that even the LS students may not be completely free from signaling in their major choice. From this perspective, our estimate θ_{econ} is likely to understate the true magnitude of the signaling effect suffered by SS students—and thus by IM students.

Table 3 reports the maximum likelihood estimates. A few patterns are observed. First, the preference estimate $\hat{\gamma}_{econ}$ is positive and statistically significant. Although smaller in magnitude, the corresponding estimates for Political Science/IR, Psychology, and Communication are also positive and statistically significant. This means that LS students value Economics highest, followed by Political Science/IR, Psychology, and Communication, when compared against Sociology, the omitted major. The majors valued less than Sociology (by LS students) are Geography and Social Welfare. The relative rankings in terms of these coefficients are in line with the common perception of the relative popularity of the majors. Second and more important,

Table 3: Preference Estimates

		Estimate	SE
Panel A: γ_j (FE common to all students)			
	Sociology	0	
	Econ	2.589	(0.354)
	Poli Sci/IR	1.415	(0.407)
	Anthropology	-0.475	(0.575)
	Psychology	1.245	(0.382)
	Geography	-1.417	(0.788)
	Social Welfare	-1.402	(0.793)
Panel B: θ_j (additional FE for SS student)			
	Sociology	0	
	Econ	1.211	(0.555)
	Poli Sci/IR	0.633	(0.648)
	Anthropology	-1.248	(1.243)
	Psychology	-0.245	(0.614)
	Geography	0.386	(1.146)
	Social Welfare	-0.306	(1.358)
	Communication	0.084	(0.642)
Panel C: Interaction Terms			
	Regular	-0.118	(0.162)
	Freshman GPA	0.057	(0.065)
Major characteristics:	# Courses Taken in Freshman		
1 previous year average GPA	Economics	-0.088	(0.071)
	Mathematics	-0.112	(0.080)
	Liberal Arts	0.003	0.073
	Business	0.002	0.062

Note: The sample consists of 550 students who were admitted in AY 2013 to 2016 after the sample selection. $\theta_{Sociology} = \gamma_{Sociology}$ are normalized to 0. Panel C reports the coefficients on the interaction term where the first column represents the major characteristics x_j and the second column represents the student characteristics z_i that are interacted with each x_j.

the estimate of θ_{econ}—the additional valuation SS students assign to Economics—is positive and statistically significant. Specifically, the SS students value Economics nearly 47% more than the LS students do. This means that SS students are more likely to choose Economics than their LS counterparts among social science majors. As argued earlier, SS students have more exposure to signaling than LS students, so this finding is consistent with the hypothesis that signaling biases one's choice toward a popular major—Economics in this particular context. We view this as central evidence for the role played by signaling in students' major choice in the SS channel, and indirectly for the IM channel, to the extent that the signaling value in SS reflects the signaling value in the IM channel.[39,40]

3.3 Does Signaling Affect Academic Performance?

We now turn our attention to the effect of signaling by investigating students' performances subsequent to their major selection, particularly in their major courses. Our theory suggests that a signaling bias toward a popular major comes at the expense of idiosyncratic preferences and aptitude (Corollary 1), and in our empirical context, this would harm one's performance in the major courses, but not necessarily in non-major courses. Therefore, the signaling hypothesis suggests that SS students who choose Economics will perform worse in their major courses (but not in non-major courses) than their LS counterparts, and such a finding will further disfavor the selection hypothesis. To study the effect, we first take a reduced-form approach, followed by a more structural approach.

3.3.1 The Effect of Signaling on Academic Performance

We first provide a reduced-form analysis of the effects of signaling on major performance. Table 4 shows results on the following regression of Sophomore GPAs:[41]

$$y_i = \beta_0 + \beta_1 Econ_i \cdot SS_i + \beta_2 nonEcon_i + \beta_3 nonEcon_i \cdot SS_i + \gamma' z_i + e_i \tag{4}$$

where $Econ_i$ is a dummy variable that equals 1 if student i's major is Economics and 0 otherwise, and SS_i is a dummy variable that equals 1 if student i's admission channel is SS and 0 otherwise.

The dependent variables y_i are the total average GPA (Total), average liberal arts GPA (Lib Art), and social science core major average GPA (Major-core) whose scale is from 0 (F) to 4.3 (A+). Our primary interest is in core major courses, Major-core. Core major courses are the required courses that all students in a given major must take. We focus on these courses since they are the same for both LS and SS students of any given major, which facilitate a clear comparison between LS and SS. LS and SS students take exactly the same set of core

[39] Note that the estimates of θ for all other majors are not significant. This means that they are all "victims" of the signaling bias toward Economics in the statistically similar degrees as Sociology, the omitted major. That said, the estimate for Psychology is negative and the one for Political Science/IR, the second popular major, is positive, suggesting that the former fared relatively worse and the latter fared relatively better than Sociology, in their "signaling loss" to Economics.

[40] We perform a simple counterfactual analysis to quantify what the estimates mean in terms of the magnitude of signaling in Online Appendix A.3. We find that once the signaling effect exhibited by the SS students is removed, students are less likely to choose Economics and more likely to choose Psychology and Communication, suggesting that those two majors are the biggest losers of the signaling bias.

[41] Recall that all students in our sample chose their majors when they rise to their Sophomore year. Hence, Sophomore GPAs capture students' performance immediately after their major choices.

major courses, along with their IM counterparts, in their second year from the same sections taught by the same instructors. For example, Economics core major courses are Microeconomics, Macroeconomics, Economic History, Mathematics for Economics, and Introductory Statistics for Economists.

The coefficients of our interest are β_1 and β_3, which capture the relative performances of SS students in Economics and non-Economics majors, respectively, in mean differences relative to LS students. Coefficient β_0 captures the mean GPA of LS students in Economics, and β_2 captures the mean difference of non-Economics from Economics students in LS. We do not control for student observable characteristics in columns (1)-(3), giving the coefficients an interpretation of raw group differences. Columns (4)-(6) control for student observable characteristics.

Table 4: Regression of GPA

	(1) Total	(2) Lib Art	(3) Major-core	(4) Total	(5) Lib Art	(6) Major-core
Econ×SS (β_1)	-0.191	0.035	-0.411	-0.005	0.148	-0.078
	(0.057)	(0.060)	(0.091)	(0.036)	(0.016)	(0.033)
non-Econ (β_2)	0.030	-0.034	0.304	0.121	0.005	0.600
	(0.060)	(0.071)	(0.117)	(0.036)	(0.054)	(0.073)
non-Econ×SS (β_3)	-0.055	0.069	-0.204	0.024	0.106	-0.088
	(0.067)	(0.077)	(0.156)	(0.076)	(0.100)	(0.095)
Regular				-0.092	-0.066	0.060
				(0.014)	(0.040)	(0.050)
Freshman GPA				0.325	0.276	0.435
				(0.030)	(0.024)	(0.023)
# Econ. Courses in Freshman				-0.010	-0.024	0.058
				(0.007)	(0.006)	(0.007)
# Math Courses in Freshman				-0.002	-0.016	0.165
				(0.026)	(0.015)	(0.017)
# Lib. Art. Courses in Freshman				-0.011	0.018	0.046
				(0.011)	(0.017)	(0.009)
# Business Courses in Freshman				0.007	0.010	0.049
				(0.015)	(0.019)	(0.032)
Constant (β_0)	3.449	3.627	3.147	3.355	3.579	2.851
	(0.044)	(0.049)	(0.073)	(0.032)	(0.017)	(0.021)
Observations	550	545	463	550	545	463

Note: Estimates for (4) are reported. Columns (1)-(3) do not control for student observable characteristics, and columns (4)-(6) are the full model in ((4)) with the same set of student observables as in Table 3 controlled for. Robust standard errors are reported in columns (1)-(3), and clustered standard errors at the major level are reported in columns (4)-(6).

Several observations are made. First, β_1, the coefficient on the interaction term is negative with statistical significance when the dependent variable is the *core major* GPA (columns (3) and (6)), as expected. The result shows that all else equal, SS students majoring in Economics suffer a GPA loss of 0.078 in core major courses, compared with the LS students majoring in Economics. This loss is significant; it accounts for nearly 10% of the standard deviation of GPA in core major courses, which is about 0.8. Namely, the higher exposure by SS Economics students to signaling than LS Economics students resulted in a relatively more adverse selection of major fit/aptitude for the former students, which adversely impacted their performances in the core major courses. As will be discussed in Section 3.4, this finding is inconsistent with a selection hypothesis that SS students might have been positively selected in terms of unobserved aptitude in Economics.

An alternative hypothesis may be that SS students are more poorly selected in comparison with LS students in the college admission stage. This hypothesis is made implausible by the next two findings.

Second, β_1 is actually positive (and significant at 1% in column (5)) when the dependent variable is *Liberal Arts* GPA. In other words, SS Economics students perform better on average than LS Economics students in the Liberal Arts courses. A natural interpretation is that, due to signaling, the Economics major attracts SS students who have relatively stronger aptitudes toward non-Economics social science majors, which one may argue are closer to Liberal Arts courses than Economics core major courses. This result, together with the first observation above, lends support to the views that signaling, or prestige consideration, significantly influences a student's major selection and the associated bias in terms of a student's major fit affects students' performances in both their core major and Liberal Arts courses.

Finally, the coefficient β_3, which captures the effect of signaling for non-Economics major students on core major course performance, is negative but not significantly different from zero (see columns (3) and (6)), meaning that there is no evidence that LS students perform better on average than SS students in non-Economics major core courses.

3.3.2 The Effect of Major Fit on Academic Performances

We next ask to what extent a student's idiosyncratic preference/aptitude toward her chosen major contributes to her academic performance. Answering this question will help us to pin down the source of and to quantify the academic losses associated with signaling established in Section 3.3.1, which in turn will help us to identify the nature of the welfare cost of signaling. But generally, the answer will inform students about how their major fit/aptitude matters for their academic success in their chosen field and thus help to guide their major selection.

To proceed, we first compute the expected value of the major fit for each chosen major. To this end, rewrite the utility of student i in major j in (3) as

$$U_{ij} = \gamma_j + \theta_j SS_i + \sum_l \delta^l x_j^l z_i^l + \varepsilon_{ij} = V_{ij} + \varepsilon_{ij}.$$

We then compute the so-called *control function* for each major j for student i:

$$\lambda_{ij} = E[\varepsilon_{ij} - \mu | x_j, z_i, j(i)] = E[\varepsilon_{ij} | V_i, j(i)] - \mu,$$

where $V_i = (V_{i1}, \cdots, V_{iJ})$ and μ is the Euler–Mascheroni constant which is the unconditional mean of ε_{ij}. In words, λ_{ij} measures the conditional expectation of student i's idiosyncratic preference for j conditional on choosing $j(i)$ for her major, adjusted by its unconditional mean. Henceforth, we shall call $\lambda_{ij(i)}$—control function evaluated at the chosen major $j(i)$— student i's **(unobservable) chosen major fit**.[42]

We are now in a position to study the role played by a student's major fit in her academic performance. We consider the following linear projection of some potential outcome Y_{ij} on major specific intercept α_j, student observable characteristics z_i and the (unobservable) major fit ε_{ij}:

$$Y_{ij} = \alpha_j + \beta z_i + \varphi \cdot (\varepsilon_{ij} - \mu) + e_{ij} \tag{5}$$

where Y_{ij} is student i's potential GPA from the courses she takes in major j and e_{ij} is simply a projection error. The observed GPA is $Y_i = \sum_j 1\{j(i) = j\} Y_{ij}$. It is convenient to view (5) as the GPA production function of major j on the major fit. Of particular interest for our purpose is φ, the dependence of a student's potential academic performance on her major fit ε_{ij}, the unobserved idiosyncratic taste defined in the major choice utility equation (3).[43]

(5) cannot be directly estimated since ε_{ij} is not observed and we only observe Y_{ij} for the chosen major $j = j(i)$. Taking conditional expectations, the mean observed outcome at major j is given by:

$$E[Y_i | x_j, z_i, j(i) = j] = \alpha_j + \beta z_i + \varphi \cdot \lambda_{ij}. \tag{6}$$

One would expect φ to be positive; namely, one's major fit contributes to her performance on major courses. Indeed, this is what we find in the OLS regression of (6).

Table 5 shows the regression results of (6) in which we allow the return to a major fit to

[42] Following Dubin and McFadden (1984), one can derive $\lambda_{ij(i)} = -\log(P_{ij(i)})$ where $P_{ij} = \frac{\exp(V_{ij})}{\sum_{k \in \mathscr{J}} \exp(V_{ik})}$ is the probability that j is chosen out of the choice set $\mathscr{J} := \{1, 2, \cdots, J\}$. See Online Appendix A.4 for the summary statistics of the chosen major fit.

[43] From this perspective, the role of the control function here is somewhat different from that often found in value-added estimation. In such an exercise (see e.g., Abdulkadiroğlu et al., 2020; Otero, Barahona and Dobbin, 2021), the control functions are added to control for possible unobserved omitted variables that may affect the assignment to the evaluated treatment. Here, we view the control function of the chosen major as the independent variable of primary interest rather than as a controlling variable. Relatedly, we do not include the control functions for unchosen majors since there is no theoretical ground for them to affect the GPA for the chosen major and, no less importantly since our sample size is not big enough to power them. One may still argue that the magnitude of the chosen major fit *relative to* unchosen major fits is what matters, and relatedly, in Appendix A.5, we report the regression results of an alternative version of (6) in which we replace $\lambda_{ij} = \lambda_{ij(i)}$ by its (individual specific) demeaned version $\bar{\lambda}_{ij} := \lambda_{ij} - \frac{1}{J} \sum_{k=1}^{J} \lambda_{ik}$. We do not find significantly different results.

be different between Economics and non-Economics majors. Of particular interest is column (3) which shows that the return to a major fit for Economics core major courses, φ_{Econ}, is estimated to be 0.283 with statistical significance at 5% (standard error 0.098). More precisely, this means that when an Economics student's major fit increases by 1 standard deviation of its unconditional distribution, her Economics core major GPA increases on average by 0.363. This amounts to 45% of a standard deviation of GPA in core major courses, which is about 0.8. This suggests a significant role played by a major fit toward a student's academic performance in Economics. An implication is that a student contemplating majoring in Economics must consider her major fit for Economics seriously at least from the perspective of academic success.

Table 5: GPA on Major Fit for a Chosen Major

	(1) Total	(2) Lib Art	(3) Major-core
Major Fit×Econ	0.039	-0.340	0.283
	(0.060)	(0.059)	(0.098)
Major Fit×non-Econ	0.102	0.189	-0.181
	(0.051)	(0.082)	(0.177)
Regular	-0.093	-0.060	0.069
	(0.025)	(0.047)	(0.051)
Freshman GPA	0.324	0.276	0.437
	(0.030)	(0.025)	(0.022)
# Econ. Courses in Freshman	-0.010	-0.030	0.062
	(0.005)	(0.004)	(0.007)
# Math Courses in Freshman	-0.000	-0.020	0.166
	(0.020)	(0.013)	(0.020)
# Lib. Art. Courses in Freshman	-0.011	0.019	0.058
	(0.011)	(0.018)	(0.021)
# Business Courses in Freshman	0.007	0.011	0.027
	(0.015)	(0.019)	(0.022)
Observations	550	545	463

Note: We allow φ to differ between Economics and non-Economics majors. Major specific intercepts ($\hat{\alpha}_j$) are omitted, and and standard errors are clustered at the chosen major level.

Interestingly, the fit for Economics predicts poor performance in Liberal Arts courses, with an equally sizable estimate of -0.340 (with statistical significance at 1%). This may reflect the unique nature of Economics in comparison with other humanities and social science disciplines in terms of its methodology and style. Also interestingly, almost the opposite patterns are observed with the other social science majors. For non-Economics major students, a student's major fit is no longer a significant predictor of her academic success at core major courses but, unlike an Economics major, it is a significant predictor of her success at Liberal Arts courses.

The patterns so far appear to indicate the special nature of the major fit required for

Table 6: GPA on Major Fit for Economics

	(1) Major-core	(2) Major-core,math	(3) Major-core,not Math
Major Fit	0.219	0.884	-0.163
	(0.312)	(0.397)	(0.371)
Regular	0.035	0.128	-0.056
	(0.090)	(0.122)	(0.096)
Freshman GPA	0.451	0.406	0.476
	(0.038)	(0.046)	(0.049)
# Econ. Courses in Freshman	0.066	0.026	0.053
	(0.041)	(0.064)	(0.047)
# Math Courses in Freshman	0.160	0.171	0.113
	(0.043)	(0.057)	(0.053)
# Lib. Art. Courses in Freshman	0.038	0.003	0.013
	(0.042)	(0.055)	(0.048)
# Business Courses in Freshman	0.015	0.056	-0.004
	(0.032)	(0.031)	(0.045)
Observations	351	286	329

Note: We report results on the regression of (6) restricting to Economics major students. Column (1) uses the Economics core major courses as the dependent variable, column (2) uses only the Economics core major courses related to math (Mathematics for Economics, Introductory Statistics for Economists), column (3) uses only those not related to math (Microeconomics, Macroeconomics, Economic History). Robust standard errors are reported in parentheses.

Economics vis-a-vis those required for other social science majors. In Table 6, we separately regress GPAs for Economics core major courses that are math-oriented (Mathematics for Economics, Introductory Statistics for Economists) and those that are not (Microeconomics, Macroeconomics, Economic History). We find that the positive association between Economics major core GPA and econ major fit is largely driven by a strong positive association between the math-oriented Economics major core GPA and the econ major fit in column (2). In fact, the association is indistinguishable from 0 for non-math-oriented courses in column (3). These two facts support the hypothesis that the aptitude for math constitutes a crucial element of the fit for Economics.

Using Table 5, we can piece together a picture of how a student's pursuit of prestige forces her to sacrifice her major fit and ultimately her academic success in terms of her GPA. In Appendix A.4, we regress the chosen major fit on the same covariates as in (4) and find that relative to LS students, SS students on average sacrifice their major fits for Economics by 0.325, due to prestige concerns. These losses of major fit translate via column (3) of Table 5 into average GPA losses of $0.325 \times 0.283 = 0.092$ on core major courses. This core major GPA loss due to the loss of major fit almost reproduces the core major GPA loss of 0.078 GPA found in column (6) of Table 4, which we view as supporting the hypothesis that the sacrificing of a major fit constitutes an important source of inefficiency associated with signaling.

3.4 The Selection of Students into SS and LS

Here we address several possible hypotheses regarding the selection of students into SS and LS that may jeopardize our interpretation of the findings.

First, one may worry that SS students are selected to be biased toward Economics in their intrinsic preferences compared with LS students, meaning that when they applied to CSS, the former group already had planned to select Economics as their major, for reasons unrelated to signaling. If this were the case, then those with a stronger interest in Economics would be more likely to enter SS than LS. This type of selection would bias the estimate of θ_{econ} in the direction of overstating the true value of signaling. While the available data do not permit us to directly disprove this hypothesis, we do not find it plausible. If such selection were important, then one would expect Economics majors from SS to perform better than their LS counterparts in Economics major courses. However, we find the opposite to be true. Namely, Table 4 shows that the Economics majors from SS performed worse in Economics major courses than their LS counterparts subsequent to their major choices. We thus find this selection hypothesis implausible.

Second, one may argue that students who prefer non-Economics social science majors may have selected LS to avoid the stigma associated with those majors. Note this hypothesis is consistent with our theory—namely that prestige concerns are important for students' major choices. Strictly speaking, however, such selection would entail a similar bias for our estimate of θ_{econ} as the first hypothesis, if one interprets it as signaling made in the second-year major choice; rather the estimate would additionally reflect the signaling made in their initial choice between SS and LS. Nevertheless, this hypothesis is also inconsistent with our findings in Section 3.3. If the hypothesis were true, one would expect non-Economics majors from LS to perform better in their major courses than their SS counterparts. And similarly, one would expect the opposite pattern to hold for Economics major courses. However, as shown in Table 4, the performance in major courses by the non-Economics majors from LS is not statistically different from that of the SS counterparts, and the Economics majors from LS performed better in Economics major courses than their SS counterparts.

Finally, LS students' major choices were not restricted to social science majors, whereas SS students could choose only from social science majors. One may worry that this difference may lead the LS students in our sample (those who chose social science majors) to be more positively selected in terms of aptitude for social science majors than their SS counterparts, leading to a better performance by LS students in the Economics majors in Table 4. Of course, this effect is presumably offset by a similar selection that would have occurred for our SS students at the time they applied to CSS. Nevertheless, this possible selection would affect the interpretation of our estimates. However, this hypothesis is also inconsistent with the findings of Section 3.3. If such selection were significant, LS students would perform uniformly better in *all* social science majors. As discussed above, Table 4 shows that while Economics majors from LS performed better in their major courses, non-Economics majors from LS did not perform better compared with their SS counterparts.

4 Related Literature

The current paper is related to several strands of literature. On the theory side, we build on Spence (1973)'s signaling model to study prestige seeking in college applications. As mentioned, our focus is on students' interactive and competitive signaling behavior and its implications for the allocation of idiosyncratic program fits and social welfare. Similarly to the current paper, MacLeod and Urquiola (2015) studies a model in which the signaling motive entails a hierarchical sorting of applicants into colleges.[44] While the reputational sorting/selection into colleges is similar, they are concerned about different behavioral and welfare implications. MacLeod and Urquiola (2015) consider students who are ex-ante homogeneous in their preferences and abilities and focus on the moral hazard problem in which students overinvest in college test preparation and underinvest in studying after admission. By contrast, we consider students who are heterogeneous in academic abilities and program fits and study how the prestige concerns entail misallocation of students' fits for academic programs. We view the two approaches as mutually compatible and complementary. Similar to us, Avery and Levin (2010), Rothschild and White (1995), Epple and Romano (1998), and Epple, Romano and Sieg (2006) consider sorting of agents based on their heterogeneous abilities and preferences, but they do not study prestige seeking behavior.

The current paper is motivated by a large and growing body of evidence suggesting that the graduates of elite colleges enjoy a significant wage premium that cannot be explained by the value added for students of similar qualities.[45] In particular, in the South Korean context, Kim and Kim (2012) find that 'elite college premium' exists in the South Korean labor market—namely, many respondents in the Korean Labor & Income Panel Study (KLIPS) experienced

[44] MacLeod et al. (2017) provide empirical support to the assumption that employers use college reputation (defined as graduates' mean admission scores) to set wages.

[45] See Brewer, Eide and Ehrenberg (1999), Dale and Krueger (2002), Hoekstra (2009), Li et al. (2012), Zimmerman (2014), MacLeod et al. (2017), Ge, Isaac and Miller (2018), Canaan and Mouganie (2018), Zimmerman (2019), Sekhri (2020), Bleemer and Mehta (2020), Bleemer (2021), and Jia and Li (2021).

discrimination in employment, promotion, and wage based on the ranking of colleges graduated. We explore the signaling implications of such premium for students' choice of programs in college applications and provide evidence of the signaling concerns and their implications for academic performances.

In that regard, the current paper also contributes to the empirical literature that provides evidence for Spencian signaling. Lang and Kropp (1986) and Bedard (2001) provide empirical evidence in favor of the signaling hypothesis using variations in compulsory attendance laws or university access. In a similar vein, Bostwick (2016) shows evidence for signaling behavior using the choices of STEM majors by students at non-elite colleges. While similar in the general theme, our empirical analysis is distinguished by its setting (competitive major selection in the IM system) and the structural approach. Also, the effect of signaling on students' academic performance has no analogs in the previous literature.

Finally, the current paper contributes to the understanding of major choices (see Altonji, Arcidiacono and Maurel, 2016, for a survey), in particular the difference between two systems—IM and DM—with regard to college major choice (Malamud, 2010; Bordon and Fu, 2015). Bordon and Fu (2015) compare the systems focusing on the trade-offs associated with uncertainty students face on their major fits in the IM and the lack of peers sharing the same majors in the DM; their counterfactual analysis suggests a modest benefit from switching to the DM system largely due to the reduced uncertainty on major fits. We take an orthogonal and complementary approach focusing on the role played by the signaling in the major choice under IM and its impact on student's academic performances and arrive at a similar conclusion—namely, that DM would improve welfare by eliminating signaling distortion in major choice and improve the academic performance in major courses.

5 Conclusion: Discussions and Policy Implications

We conclude here by discussing additional implications of prestige seeking particularly on group inequality. We then discuss several policy interventions that may mitigate the negative effect of prestige-seeking. The details of the analysis supporting the discussions are available in the online appendix.

Distributional consequences of prestige concerns. A troubling concern in higher education is the disparate access to elite colleges by students with differing parental incomes (e.g., Chetty et al., 2020). A natural question is how the prestige concern affects this disparity. In the Online Appendix B.1, we consider an extension in which there are two groups of students: the *privileged* and the *underprivileged*. The former group has a score distribution that first-order stochastically dominates that of the latter group, presumably because the former group has greater access to test-taking preparation services than the latter group.

The analysis shows that prestige seeking reduces the underprivileged group's access to the elite program (i.e., program A) and lowers their utilitarian welfare disproportionately. Specifically, as the prestige concerns increase from τ (possibly equal to zero) to $\tau' > \tau$, the enrollment share of the underprivileged in the "elite" program falls and their utilitarian welfare loss from program mismatches worsens more severely than that of the privileged. This result is intuitive. With increased prestige concerns, more high-ability students switch to the elite program ignoring their program fits. This makes the elite program more selective, causing its cutoff to rise. This makes the elite program less available to students, including those with a high fit with the program. This process, as in the baseline analysis, creates more mismatches, but the underprivileged suffer disproportionately larger mismatches due to their disadvantaged access to test-taking resources.

Immediate major choice versus deferred major choice. Our analysis provides a strong argument in favor of DM over IM. The competitive screening associated with IM creates prestige-seeking in major choices and causes welfare loss associated with major mismatches. By allowing students to choose their majors freely, DM mitigates such mismatches. The LS system in SNU introduced in 2009 has been a successful experiment in this regard.[46] Our empirical analysis in Section 3.3 reinforces this assessment. We have shown that the LS students majoring in Economics performed significantly better than their SS counterparts in core major courses.

While a switch from IM to DM may reduce the prestige gap across majors, this may exacerbate the prestige gap across colleges since students may shift their signaling effort to a college choice from a major choice. Indeed, casual observation suggests that, in countries such as the US where DM is adopted, elite colleges command much more prestige than elite majors. One may then worry that DM may entail mismatches in college choices, just as IM entailed mismatches in major choices. While this is a valid concern, major mismatches are arguably more consequential than college mismatches from the human resource allocation perspective. Online Appendix B.2 indeed confirms this intuition, showing that DM is more likely to be superior to IM when college fits are negligible.[47]

[46]The LS college system adopted in other universities in Korea such as Yonsei university, arguably the second best in Korea, was not met with similar successes. In the case of Yonsei, unlike SNU, sufficient safeguards were not made to separate the identities of LS students from the IM students; in a sense, their treatment was similar to that of the SS students of SNU. Consequently, the LS students in these colleges were subject to significant prestige concerns, in fact too significant for unpopular majors to take viable footholds.

[47]In addition, DM also enables students to explore and learn their own major fits and preferences during the pre-major phase of the college career (for example, see Malamud, 2010; Bordon and Fu, 2015).

Signal accuracy. To the extent that signaling rests on a program's screening of students' abilities based on their test scores, one suspects that signal accuracy of the score would affect the incentive for prestige seeking. Intuitively, the more accurate the "scores" (used by programs to screen applicants) are in reflecting applicants' abilities, the more severe the prestige concerns will be. This insight is formally confirmed in our analysis in Online Appendix B.3.

In this extension, the signal accuracy is represented by the distribution of the posterior means of applicants' abilities. We show that if the signal accuracy increases in the sense of *supermodular precision*,[48] then a prestige gap increases and utilitarian welfare decreases. Indeed, with increased signal accuracy, programs' screening becomes more informative about students' abilities. So, the applicants become more willing to sacrifice their program fits to seek prestige.

This result points to one avenue in which policymakers may ameliorate the deleterious effect of prestige-seeking. They can coarsen the measures of applicants' abilities made available to colleges. Coarsened performance measures—"pass" or "fail," for instance—are widely used in a variety of contexts. While coarsening the scores of standardized tests such SAT will require coordination among colleges or a centralized regulation by a higher authority, this is not without precedents. The Korean government mandated in 2008 a grading system that coarsens the raw CSAT score into 9 categories, precisely to mitigate the competition in prestige seeking.[49]

Restricting application. College application systems vary in terms of the set of choices available to applicants. In the US, the advent of CommonApps dramatically expanded the number of colleges one can apply to at reasonable financial costs and efforts. Meanwhile, selective colleges in the US limit the application to a single choice for Early Admissions. Restricted choice, for instance, in the context of Early Admissions, has been rationalized as credibly eliciting applicants' idiosyncratic preferences for colleges (Avery and Levin, 2010) or as easing congestion and yield management for colleges (Che and Koh, 2016), when the applicants are

[48]More precisely, signal F_1 is *more supermodular precise* than signal F_2 if for $1 \geqslant c' \geqslant c \geqslant 0$:

$$F_1^{-1}(c') - F_1^{-1}(c) \geqslant F_2^{-1}(c') - F_2^{-1}(c)$$

which in our setup with unbiased signals is equivalent to

$$\mathbb{E}_1\left[\theta\,\middle|v = F_1^{-1}(c')\right] - \mathbb{E}_1\left[\theta\,\middle|v = F_1^{-1}(c)\right] \geqslant \mathbb{E}_2\left[\theta\,\middle|v = F_2^{-1}(c')\right] - \mathbb{E}_2\left[\theta\,\middle|v = F_2^{-1}(c)\right]. \tag{7}$$

In words, the more supermodular precise signal, F_1, has a (normalized) conditional expectation function that is more sensitive to changes in c than the less sensitive F_2 at every c (see Shaked and Shanthikumar, 2007). We note that our result does not hold when comparing signal precision using standard notions such as mean preserving spread.

[49]However, the mandate was eventually retracted after one year due to political pushbacks. For related Korean news articles, see https://www.donga.com/news/article/all/20040826/8099681/1, https://www.chosun.com/site/data/html_dir/2008/01/23/2008012300057.html.

uncertain about their admission chances. In the current context, restriction on the application may alleviate prestige concerns and the associated program mismatches. Online Appendix B.4 studies a model in which students do not know their scores when applying to a program and shows that restricting the application to one program induces a lower prestige gap and higher utilitarian welfare compared to when there is no such restriction. The basic intuition is that, unlike unrestricted application, restricted application imposes a risk that when one fails to get into the prestigious program, she may lose admission *even to a lesser* program. This extra risk makes students more cautious in trading off the program fits in pursuit of prestige. Consequently, the application decision is steered more toward one's program fits and away from prestige seeking, which in turn lessens the signaling content of prestige, leading to an advantageous de-amplification of prestige seeking. The process yields an unambiguous reduction in program mismatches.[50]

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[50]From this perspective, it is perhaps no surprise that countries adopting IM often place restrictions on the number of programs students may apply to. For example, IM applicants can only apply to 3 colleges in Korea, 3 public colleges and other private colleges (with different exam dates) in Japan, and 10 college-major pairs in France.

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Appendix.

A Proofs for Section 2

To prove the results in Section 2, we first need to construct the fixed-point operator $\phi : [0,1] \to [0,1]$. To this end, we fix any $\delta \in [0,1]$ and characterize the resulting assignment by cutoffs $\hat{\alpha}(\delta)$ and $\hat{v}_A(\delta)$.

First, we observe that each student would prefer A over B as long as her idiosyncratic preference for B relative to A—i.e., $\varepsilon_B - \varepsilon_A = -\alpha$—does not exceed the quality gap Δ plus $\tau\delta$. Consequently, the preference cutoff $\hat{\alpha}(\delta)$ for program A must satisfy $-\hat{\alpha}(\delta) = \min\{\Delta + \tau\delta, 1\}$,[1] or

$$\hat{\alpha}(\delta) = \max\{-\Delta - \tau\delta, -1\}. \tag{8}$$

Recall the types of students enrolling in A is then given by:

$$T_A(\delta) := \{t = (\alpha, v) \,|\, \alpha \geqslant \hat{\alpha}(\delta) \text{ and } v \geqslant \hat{v}_A(\delta)\}, \tag{9}$$

[1] Recall that 1 is the upper bound for the range of $-\alpha$.

where $\hat{v}_A(\delta)$ is the score cutoff for A. Hence, given $\hat{\alpha}(\delta)$, the market clearing condition for A pins down the score cutoff $\hat{v}_A(\delta)$ by:

$$(1 - G(\hat{\alpha}(\delta)))\,(1 - F(\hat{v}_A(\delta))) = \kappa_A. \tag{10}$$

The cutoffs $(\hat{\alpha}(\delta), \hat{v}_A(\delta))$ thus determined in turn induce the prestige value of A:

$$\mathbb{E}_A[v] = \mathbb{E}[v|t \in T_A(\delta)] = \frac{\int_{\hat{v}_A(\delta)}^1 v dF(v)}{1 - F(\hat{v}_A(\delta))} =: e(\hat{v}_A(\delta)). \tag{11}$$

and the total prestige value of the two programs:

$$\kappa_A \mathbb{E}_A[v] + \kappa_B \mathbb{E}_B[v] = \int_{\underline{v}}^1 v dF(v) = (1 - F(\underline{v}))e(\underline{v}). \tag{12}$$

Combining (11) and (12) yields a new prestige gap:

$$\phi(\delta) := \mathbb{E}_A[v] - \mathbb{E}_B[v] = \frac{\kappa_A + \kappa_B}{\kappa_B}(e(\hat{v}_A(\delta)) - e(\underline{v})), \tag{13}$$

which completes the construction of the map ϕ.

Proof of Lemma 1. Consider an equilibrium assignment $\mathfrak{m}$ with prestige gap $\hat{\delta}$ and cutoff scores $\hat{v}_A \geqslant \hat{v}_B$ that satisfy conditions (i) and (ii). We need to show that $\hat{\delta} = \phi(\hat{\delta})$. First, it is clear that $\hat{v}_B = \underline{v}$. Also, for each $t = (\alpha, v)$ with $v \geqslant \hat{v}_A$, condition (i) requires $\mathfrak{m}(t) = A$ if and only if $\alpha \geqslant \hat{\alpha}(\hat{\delta})$, which in turn implies $\hat{v}_A(\hat{\delta}) = \hat{v}_A$ by condition (ii). Given the cutoffs $\hat{v}_A = \hat{v}_A(\hat{\delta})$ and $\hat{v}_B = \underline{v}$, $\mathbb{E}_A[v]$ and $\mathbb{E}_B[v]$ can be obtained via (11) and (12). Thus, $\phi(\hat{\delta}) = \mathbb{E}_A[v] - \mathbb{E}_B[v] = \hat{\delta}$ since $\hat{\delta}$ is the equilibrium prestige gap.

To prove the converse, assume that $\hat{\delta}$ is a fixed point of mapping ϕ. Starting with $\hat{\delta}$, let us define $\hat{\alpha}(\hat{\delta})$, $T_A(\hat{\delta})$ and $\hat{v}_A(\hat{\delta})$ as in (8), (9) and (10), and set $\hat{v}_A := \hat{v}_A(\hat{\delta})$ and $\hat{v}_B = \underline{v}$. Define assignment $\mathfrak{m}$ as $\mathfrak{m}(t) := A$ for all $t \in T_A(\hat{\delta})$ while $\mathfrak{m}(t) := B$ for all types with $v \geqslant \underline{v}$ that are not in $T_A(\hat{\delta})$ and $\mathfrak{m}(t) := \varnothing$ for all other types (i.e., for types with $v < \underline{v}$). By construction, $(\hat{v}_A, \hat{v}_B)$ satisfy conditions (i) and (ii) in our definition of equilibrium assignment. $\square$

Proof of Theorem 1. The existence of equilibrium follows from establishing that the self-map ϕ defined via (8),(10), and (13) is monotonic and thus has a fixed point.

Let us first prove that ϕ is a self-map. Consider any $\delta \in [0, 1]$ and let $\delta' = \phi(\delta)$. Since $\Delta, \tau \geqslant 0$, we have $\hat{\alpha}(\delta) \leqslant \max\{-\Delta, -1\}$. Using this observation and $1 - G(-\Delta) \geqslant \frac{\kappa_A}{\kappa_A + \kappa_B}$, we have

$$1 - G(\hat{\alpha}(\delta)) \geqslant 1 - G(\max\{-\Delta, -1\}) \geqslant \frac{\kappa_A}{\kappa_A + \kappa_B}.$$

By this and (10), we have $1 - F(\hat{v}_A(\delta)) \leqslant \kappa_A + \kappa_B = 1 - F(\underline{v})$, which implies $\hat{v}_A(\delta) \geqslant \underline{v}$ and thus $\delta' = \phi(\delta) = \frac{\kappa_A + \kappa_B}{\kappa_B}(e(\hat{v}_A(\delta)) - e(\underline{v})) \geqslant 0$ since $e(\cdot)$ is increasing. Also, $\delta' \leqslant 1$ is immediate from the fact that $\delta' = \phi(\delta) = \mathbb{E}_A[v] - \mathbb{E}_B[v]$ and that $\mathbb{E}_A[v] \leqslant 1$ and $\mathbb{E}_B[v] \geqslant 0$ (since they are the average scores of student types in A and B). Next, to prove the monotonicity of ϕ, consider $\delta', \delta'' \in [0,1]$ with $\delta' < \delta''$. From (8), we have $\hat{\alpha}(\delta') \geqslant \hat{\alpha}(\delta'')$, which implies $\hat{v}_A(\delta') \leqslant \hat{v}_A(\delta'')$ by (10). Then, since $e(\cdot)$ is increasing, we have $\phi(\delta') = \frac{\kappa_A+\kappa_B}{\kappa_B}(e(\hat{v}_A(\delta')) - e(\underline{v})) \leqslant \frac{\kappa_A+\kappa_B}{\kappa_B}(e(\hat{v}_A(\delta'')) - e(\underline{v})) = \phi(\delta'')$, as desired. Given that ϕ is a nondecreasing self-map on $[0,1]$, its fixed point exists according to Tarski's fixed-point theorem. □

Proof of Proposition 1. For (i), it suffices to show that $\delta = 0$ is a fixed point of ϕ. To do so, note first that $\Delta = \delta = 0$ implies $\hat{\alpha}(\delta) = 0$ from (8) and thus $1 - G(\hat{\alpha}(\delta)) = 1/2$. This implies by (10) that $1 - F(\hat{v}_A(\delta)) = 2\kappa_A = \kappa_A + \kappa_B$, so $\hat{v}_A(\delta) = \underline{v}$ and thus $\phi(\delta) = \frac{\kappa_A+\kappa_B}{\kappa_B}(e(\hat{v}_A(\delta)) - e(\underline{v})) = 0$.

To prove (ii), we establish that $\phi'(0) > 1$ if $\tau > \bar{\tau}$, which will imply that $\phi(\delta) > \delta$ for δ close to 0. Since $\phi(1) \leqslant 1$, we must have another fixed point $\delta \in (0,1]$ of ϕ with corresponding $\hat{v}_A > \underline{v}$ and $\hat{\alpha} < 0$. To show that $\phi'(0) > 1$ if $\tau > \bar{\tau}$, observe first that with $\hat{\alpha}(0) = 0$ and $\hat{v}_A(0) = \underline{v}$. Substituting these into (14) and noting that $1 - G(0) = 1/2$, and $\frac{\kappa_A+\kappa_B}{\kappa_B} = 2$, we obtain

$$\begin{aligned}\phi'(0) &= \frac{2\tau g(0)\left(-\underline{v}(1-F(\underline{v})) + \int_{\underline{v}}^1 v dF(v)\right)}{(1-G(0))(1-F(\underline{v}))} \\ &= 4\tau g(0)\left(-\underline{v} + \frac{\int_{\underline{v}}^1 v dF(v)}{1-F(\underline{v})}\right) = 4\tau g(0)\left(e(\underline{v}) - \underline{v}\right),\end{aligned}$$

which is greater than 1 if (and only if) $\tau > \bar{\tau}$. □

Proof of Proposition 2. Observe first that, by (8), $\hat{\alpha}(0) = \max\{-\Delta, -1\} < 0$, which implies $\hat{v}_A(0) > \underline{v}$. To see this, suppose for contradiction that $\hat{v}_A(0) \leqslant \underline{v}$. We have

$$\begin{aligned}\kappa_A &= (1 - G(\max\{-\Delta, -1\}))\,(1 - F(\hat{v}_A(0))) \\ &\geqslant (1 - G(\max\{-\Delta, -1\}))(1 - F(\underline{v})) \\ &= (1 - G(\max\{-\Delta, -1\}))(\kappa_A + \kappa_B),\end{aligned}$$

where the first equality holds by (10) and the last equality by definition of $\underline{v}$. This contradicts the assumption that $1 - G(-\Delta) > \frac{\kappa_A}{\kappa_A+\kappa_B}$.

Thus, $\phi(0) = \frac{\kappa_A+\kappa_B}{\kappa_B}(e(\hat{v}_A(0)) - e(\underline{v})) > 0$. This means that $\hat{\delta} = 0$ cannot arise in equilibrium. By Theorem 1, there must exist an equilibrium with $\hat{\delta} > 0$.

To prove the uniqueness of such an equilibrium, let us first establish the following claim:

Claim 1. *If g is nondecreasing in $[-1, 0]$, then ϕ is strictly concave for $\delta \in [0, \frac{1-\Delta}{\tau})$ and constant for $\delta \geqslant \frac{1-\Delta}{\tau}$.*

Proof. Consider first $\delta < \frac{1-\Delta}{\tau}$, in which case $\hat{\alpha}(\delta)$ is equal to $-\Delta - \tau\delta$. Substituting this into (10) and applying the implicit function theorem, we obtain

$$\frac{d\hat{v}_A(\delta)}{d\delta} = \frac{\tau g(-\Delta - \tau\delta)(1 - F(\hat{v}_A(\delta)))}{(1 - G(-\Delta - \tau\delta))f(\hat{v}_A(\delta))}.$$

Letting $\hat{\alpha} = -\Delta - \tau\delta$ and $\hat{v}_A = \hat{v}_A(\delta)$ (to simplify notation), we obtain by the chain rule

$$\begin{aligned}
\phi'(\delta) &= \frac{\kappa_A + \kappa_B}{\kappa_B}\left(\frac{de(\hat{v}_A)}{d\hat{v}_A}\right)\left(\frac{d\hat{v}_A(\delta)}{d\delta}\right) \\
&= \frac{\kappa_A + \kappa_B}{\kappa_B}\left(\frac{\tau g(\hat{\alpha})(1 - F(\hat{v}_A))}{(1 - G(\hat{\alpha}))f(\hat{v}_A)}\right)\left(\frac{-\hat{v}_A f(\hat{v}_A)(1 - F(\hat{v}_A)) + f(\hat{v}_A)\int_{\hat{v}_A}^1 v dF(v)}{(1 - F(\hat{v}_A))^2}\right) \\
&= \left(\frac{\kappa_A + \kappa_B}{\kappa_B}\right)\frac{\tau g(\hat{\alpha})\left(-\hat{v}_A(1 - F(\hat{v}_A)) + \int_{\hat{v}_A}^1 v dF(v)\right)}{(1 - G(\hat{\alpha}))(1 - F(\hat{v}_A))}. \qquad (14)
\end{aligned}$$

Note that the denominator of the expression in (14) is equal to κ_A for all δ. The numerator is (strictly) decreasing in δ since $\hat{\alpha} = -\Delta - \tau\delta$ is (strictly) decreasing in δ so $g(\hat{\alpha})$ is nonincreasing in δ, and since $\hat{v}_A = \hat{v}_A(\delta)$ is increasing in δ and $-\hat{v}_A(1 - F(\hat{v}_A)) + \int_{\hat{v}_A}^1 v dF(v)$ is decreasing in $\hat{v}_A$.[2] Hence $\phi'(\delta)$ is strictly decreasing in $[0, \frac{1-\Delta}{\tau})$.

Next, for any $\delta \geqslant \frac{1-\Delta}{\tau}$, we have $\hat{\alpha}(\delta) = -1$ and thus $\hat{v}_A(\delta)$ is also constant, which means $\phi(\delta)$ is constant as well. □

Letting $\delta_m > 0$ denote the lowest equilibrium, the property of ϕ in Claim 1 together with $\phi(0) > 0$ implies ϕ intersects the 45-degrees line from above and only once at δ_m, from which the uniqueness follows immediately.□

Proof of Theorem 2. Suppose that (Δ, τ) increases from (Δ^1, τ^1) to $(\Delta^2, \tau^2) \geqslant (\Delta^1, \tau^1)$. Let $\hat{\alpha}^i(\cdot)$, $\hat{v}_A^i(\cdot)$, and $\phi^i(\cdot)$ denote the mappings defined in (8) to (13), associated with (Δ^i, τ^i) Note that the mappings $\hat{v}_A^i(\cdot)$ and $\phi^i(\cdot)$ are nondecreasing while $\hat{\alpha}^i(\cdot)$ is nonincreasing.

To prove (i), consider first an equilibrium prestige gap $\hat{\delta}^1$ under (Δ^1, τ^1). Note that by (8), $\hat{\alpha}^2(\delta^1) \leqslant \hat{\alpha}^1(\delta^1)$ since $(\Delta^2, \tau^2) \geqslant (\Delta^1, \tau^1)$. From this and (10), we have $\hat{v}_A^2(\delta^1) \geqslant \hat{v}_A^1(\delta^1)$. Thus, we have

$$\phi^2(\hat{\delta}^1) = \frac{\kappa_A + \kappa_B}{\kappa_B}(e(\hat{v}_A^2(\hat{\delta}^1)) - e(\underline{v})) \geqslant \frac{\kappa_A + \kappa_B}{\kappa_B}(e(\hat{v}_A^1(\hat{\delta}^1)) - e(\underline{v})) = \phi^1(\hat{\delta}^1) = \hat{\delta}^1 \qquad (15)$$

since $e(\cdot)$ is increasing. Given this and $\phi^2(1) \leqslant 1$ (since ϕ^2 is a self-map on $[0, 1]$), the intermediate value theorem implies the existence of $\hat{\delta}^2 \geqslant \hat{\delta}^1$ such that $\phi^2(\hat{\delta}^2) = \hat{\delta}^2$, meaning that $\hat{\delta}^2$ is an equilibrium prestige gap under (Δ^2, τ^2).

[2] To see it, note that differentiating this expression with $\hat{v}_A$ yields $-(1 - F(\hat{v}_A)) < 0$.

Consider next an equilibrium prestige gap $\hat{\delta}^2$ under (Δ^2, τ^2). Analogous to (15), we have $\hat{\delta}^2 = \phi^2(\hat{\delta}^2) \geqslant \phi^1(\hat{\delta}^2)$. Given this and $\phi^1(0) \geqslant 0$ (since ϕ^1 is a self-map on $[0,1]$), the intermediate value theorem implies the existence of $\hat{\delta}^1 \leqslant \hat{\delta}^2$ such that $\phi^1(\hat{\delta}^1) = \hat{\delta}^1$, meaning that $\hat{\delta}^1$ is an equilibrium prestige gap under (Δ^1, τ^1).

The proof of (ii) is provided in the text. □

Proof of Corollary 1. As before, let $\hat{\alpha}^1$ and $\hat{v}_A^1$ ($\hat{\alpha}^2$ and $\hat{v}_A^2$, resp.) denote the preference and score cutoffs before (after, resp.) the change. By Theorem 2-(i), for any equilibrium $(\hat{\alpha}^1, \hat{v}_A^1)$, one can find an equilibrium $(\hat{\alpha}^2, \hat{v}_A^2)$ satisfying $\hat{\alpha}^2 \leqslant \hat{\alpha}^1$ and $\hat{v}_A^2 \geqslant \hat{v}_A^1$. Let T_{jk} denote the student types (α, v) who enroll in major j before the change and in major k after the change. Clearly, T_{AB} and T_{BA} must have the same measure, which we denote by m. Note that

$$T_{AB} = \{(\alpha, v) : \alpha \geqslant \hat{\alpha}^1 \text{ and } v \in [\hat{v}_A^1, \hat{v}_A^2]\};\ T_{BA} = \{(\alpha, v) : \alpha \in [\hat{\alpha}^2, \hat{\alpha}^1] \text{ and } v \geqslant \hat{v}_A^2\}. \tag{16}$$

Thus,

$$\left[1 - F(\hat{\alpha}^1)\right]\left[G(\hat{v}_A^2) - G(\hat{v}_A^1)\right] = m = \left[F(\hat{\alpha}^1) - F(\hat{\alpha}^2)\right]\left[1 - G(\hat{v}_A^2).\right] \tag{17}$$

Letting $\mathbb{E}_j^1[\varepsilon_j]$ and $\mathbb{E}_j^2[\varepsilon_j]$ denote the average fitness of students in major j with their major before and after the parameter change, we have respectively,

$$\begin{aligned}
\kappa_A \mathbb{E}_A^1[\varepsilon_A] &= \mathbb{E}\left[\varepsilon_A \cdot 1_{\{(\alpha,v)\in T_{AA}\}}\right] + \mathbb{E}\left[\varepsilon_A \cdot 1_{\{(\alpha,v)\in T_{AB}\}}\right] \\
\kappa_A \mathbb{E}_A^2[\varepsilon_A] &= \mathbb{E}\left[\varepsilon_A \cdot 1_{\{(\alpha,v)\in T_{AA}\}}\right] + \mathbb{E}\left[\varepsilon_A \cdot 1_{\{(\alpha,v)\in T_{BA}\}}\right].
\end{aligned}$$

Thus,

$$\begin{aligned}
\kappa_A\left(\mathbb{E}_A^2[\varepsilon_A] - \mathbb{E}_A^1[\varepsilon_A]\right) =& \mathbb{E}\left[\varepsilon_A \cdot 1_{\{(\alpha,v)\in T_{BA}\}}\right] - \mathbb{E}\left[\varepsilon_A \cdot 1_{\{(\alpha,v)\in T_{AB}\}}\right] \\
=& \int_{\hat{\alpha}^2}^{\hat{\alpha}^1} \mathbb{E}[\varepsilon_A|\alpha]dF(\alpha)\left[1 - G(\hat{v}_A^2)\right] - \int_{\hat{\alpha}^1}^{1} \mathbb{E}[\varepsilon_A|\alpha]dF(\alpha)\left[G(\hat{v}_A^2) - G(\hat{v}_A^1)\right] \\
=& m\left(\frac{\int_{\hat{\alpha}^2}^{\hat{\alpha}^1} \mathbb{E}[\varepsilon_A|\alpha]dF(\alpha)}{F(\hat{\alpha}^1) - F(\hat{\alpha}^2)} - \frac{\int_{\hat{\alpha}^1}^{1} \mathbb{E}[\varepsilon_A|\alpha]dF(\alpha)}{1 - F(\hat{\alpha}^1)}\right) \\
\leqslant& m\left(\frac{\int_{\hat{\alpha}^2}^{\hat{\alpha}^1} \mathbb{E}[\varepsilon_A|\hat{\alpha}^1]dF(\alpha)}{F(\hat{\alpha}^1) - F(\hat{\alpha}^2)} - \frac{\int_{\hat{\alpha}^1}^{1} \mathbb{E}[\varepsilon_A|\hat{\alpha}^1]dF(\alpha)}{1 - F(\hat{\alpha}^1)}\right) = 0,
\end{aligned}$$

where the second equality follows from (16) and the third equality from (17) while the inequality from the fact that $\mathbb{E}[\varepsilon_A|\alpha]$ is nondecreasing in α. Thus, we have $\mathbb{E}_A^2[\varepsilon_A] \leqslant \mathbb{E}_A^1[\varepsilon_A]$, as desired. Analogously, one can show $\mathbb{E}_B^2[\varepsilon_B] \leqslant \mathbb{E}_B^1[\varepsilon_B]$. □

Online Appendix to

Prestige Seeking in College Application and Major Choice

(Not for Publication)

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A Supplementary Materials to Section 3

A.1 Data and Sample Restriction

We use confidential data acquired from the Office of Admissions of SNU.

There are three types of data. First, GPA data includes course titles, areas, letter and numeric grades and credits for all courses students took. Next, major choice data covers the major choice information of SS and LS students. Finally, demographic data includes demographic information of students, including information on gender, admission types as well as admission year. All datasets are mergeable using a scrambled student identifier.

As noted above, we focus on SS and LS students in academic years from 2013 to 2016 who chose a social science major. To ensure a large enough sample size, we pool data across multiple years. Also, we focus on those whose major choice took place just before they rose to their Sophomore years.[1] Finally, we restrict the sample to students who have GPA information in the year following major choice in order to explore the effect of signaling on major performance. In total, we have 550 students in our main analysis sample.

For interested readers, we provide summary statistics of IM students in Table A.1.

A.2 Graphical Illustration of Major Choice in SNU

The difference between SS and LS can be illustrated by Figure A.1, in which the left and right panels depict the major choice for SS and LS students respectively, and T_j represents the set of types (in terms of major fit ε) choosing alternative majors $j = A, B$ in each regime.

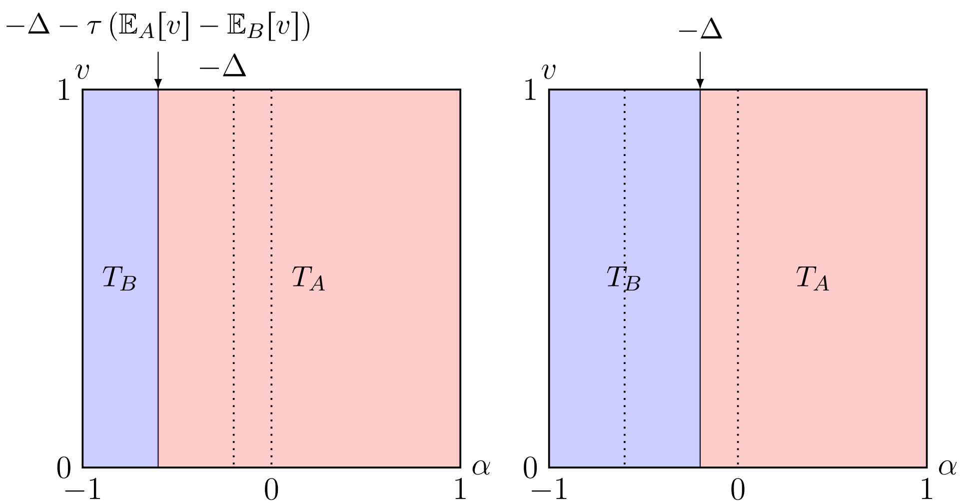
Suppose A and B correspond to popular and less popular majors, for example, Economics and Sociology, respectively. One major difference for these figures in comparison with the earlier ones for IM models (for example, Figure 1) is that since the choices are free here, there is no

[1] Though a majority of students select their major when rising to their second year, it is not mandatory and there are a small number of students who make major choices later on their academic years, when rising to their third or fourth years.

Table A.1: Student Summary Statistics: Immediate-Major (IM)

	Mean	SD
Female (%)	47.46	
Regular (%)	0.00	
Freshman Information		
GPA	3.53	0.50
# Econ. Courses	1.30	1.30
# Math. Courses	0.51	0.89
# Lib. Art. Courses	8.06	1.95
# Business. Courses	0.02	0.16

Note: The sample consists of 1064 students who were admitted in AY 2013 to 2016 through IM. 'Freshman GPA' has scales from 0 (F) to 4.3 (A+). 'Regular' is the fraction of students who were admitted through regular admissions; omitted admission methods are early admissions and other admission methods.



(a) Social Science (SS) (b) Liberal Studies (LS)

Figure A.1: SNU Student Assignment: Economics and Sociology

longer any rationing or screening. This means that in case of LS (right panel), students simply choose B if and only if $\alpha = \varepsilon_A - \varepsilon_B < q_B - q_A = -\Delta$, so the common quality difference is the only source of distortion, whereas a SS student picks B if and only if $\alpha < -\Delta - \tau\left(\mathbb{E}_A[v] - \mathbb{E}_B[v]\right)$ where $\mathbb{E}_A[v] - \mathbb{E}_B[v]$ is the prestige gap derived from the IM admissions channel.

A.3 Counterfactual Regime: with v.s. without Signaling Effects

To quantify the magnitude of signaling, we compute the aggregate probability of choosing each major based on the estimates under two scenarios: the current regime, and a counterfactual regime in which all students are LS students. Effectively, the counterfactual scenario removes the signaling effect associated with the major choice exhibited by the SS students by setting $\theta_j = 0, \forall j$.

The calculation is depicted in Figure A.2. Several features are noteworthy.

First, once the signaling effect exhibited by the SS students is removed, they are less likely to choose Economics, and more likely to choose Psychology and Communication. It suggests that the latter two majors were the biggest losers of the signaling bias toward Economics than other social science majors. As noted above, even the LS students may be exposed to major signaling; hence, major selection may change more substantially if the signaling effect were eliminated completely, say by abolishing the IM admissions altogether, which would be effectively equivalent to the system used by the US colleges.

Second, even with the signaling effect removed, a lot of students still choose Economics as their major. This reflects the high common valuation $\hat{\gamma}_{econ}$, interpreted as the high intrinsic preference for Economics.

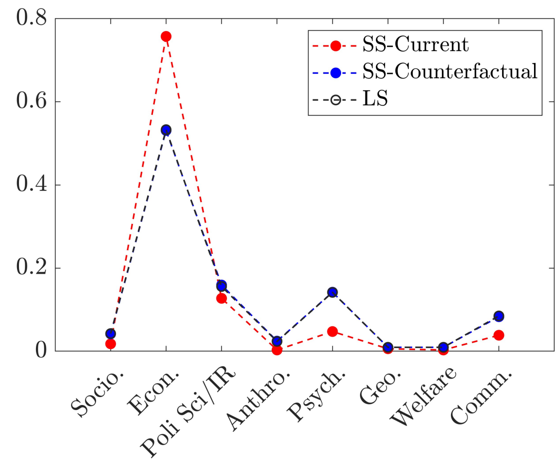


Figure A.2: Estimated Average Probability of Choosing Each Major

Note: Using (3), we compute the predicted probability of choosing each major based on the estimates under two scenarios: the current regime, and a counterfactual regime in which all students are LS students.

A.4 Chosen Major Fit

Using the preference estimates in Section 3.2, we calculate the average chosen major fit, namely the average of $\lambda_{ij(i)}$'s over students for each major and admission channel in Table A.2.

The figures "mirror" the estimates of Table 3. First of all, the fact that the chosen major fit is all positive reflects the fact that the students exercised free choice with major, which clearly yields an advantageous selection of a major fit. Second, the LS column can be explained by the estimates of γ_j's in Table 3. Namely, a more popular major (according to the non-signaling component) entails relatively more adverse selection of major fit. Specifically, Economics, which

Table A.2: Chosen Major Fit by Admission Channels

	Liberal Studies (LS)	Social Science (SS)	All
Sociology	3.317	4.176	3.660
Economics	0.622	0.275	0.381
Poli Sci/IR	1.742	1.958	1.864
Anthropology	3.668	5.789	4.022
Psychology	1.979	3.163	2.391
Geography	4.543	5.636	4.907
Social Welfare	4.543	5.636	4.907
Communication	2.545	3.360	2.887
Weighted Average	1.413	0.874	1.081

Note: The sample consists of 550 students who were admitted in AY 2013 to 2016 after the sample selection. The control functions are calculated using the main specification (3). The average across all majors within each admission channel is calculated using weights as the number of students who chose each major in each track. Note that the unconditional standard deviation of ε_{ij} is equal to $\sqrt{\pi^2/6} \approx 1.2825$.

is most popular according to Table 3, suffers most adverse selection followed by the second and third popular majors, Political Science/IR and Psychology. Finally, the SS column reflects the "signaling estimates" θ_j's in Table 3. Namely, the adverse selection for popular major—in particular, Economics—is worsened by the signaling. Equivalently, the selection is most advantageous for unpopular majors, which is consistent with the view that to overcome signaling disadvantage, one must have had a very high idiosyncratic aptitude/preference for the chosen major.

It is instructive to consider the following regressions reported in Table A.3 in relation to Table A.2:

$$\lambda_{ij(i)} = \beta_0 + \beta_1 SS_i + \gamma' z_i + e_i \tag{1}$$

$$\lambda_{ij(i)} = \beta_0 + \beta_1 Econ_i + \gamma' z_i + e_i \tag{2}$$

$$\lambda_{ij(i)} = \beta_0 + \beta_1 Econ_i \cdot SS_i + \beta_2 nonEcon_i + \beta_3 nonEcon_i \cdot SS_i + \gamma' z_i + e_i \tag{3}$$

where SS_i is a dummy variable for SS track, $Econ_i$ is a dummy variable for Economics major and $nonEcon_i$ is a dummy variable for non-Economics major, z_i is a vector of student characteristics including the same set of student characteristics as in the discrete choice model.

The result is in line with Table A.2. SS on average has lower chosen major fit than LS (column (1)), and Economics has lower chosen major fit than other majors (column (2)). Most importantly, column (3) reveals that the loss of major fit for students majoring in Economics arises from two sources: its quality premium and prestige premium. Quality premium means that Economics have higher quality ($q_{Econ} > q_{j'}, \forall j' \neq Econ$) than other majors, which is captured by LS students having higher value on Economics ($\hat{\gamma}_{Econ}$) despite absence of signaling concerns.

Table A.3: Regression of Major Fit for a Chosen Major

	(1) Major Fit	(2) Major Fit	(3) Major Fit
Econ		-2.090	
		(0.321)	
Econ×SS			-0.325
			(0.044)
non-Econ			1.645
			(0.265)
non-Econ×SS			0.462
			(0.316)
SS	-0.252		
	(0.269)		
Regular	-0.239	-0.016	0.030
	(0.134)	(0.097)	(0.058)
Freshman GPA	-0.007	0.034	0.024
	(0.041)	(0.030)	(0.025)
# Econ. in Freshman	-0.478	-0.038	-0.053
	(0.207)	(0.026)	(0.042)
# Math. in Freshman	-0.077	0.035	0.014
	(0.035)	(0.054)	(0.026)
# Lib. Art. in Freshman	0.039	0.013	0.003
	(0.056)	(0.023)	(0.022)
# Business. in Freshman	0.038	-0.006	0.009
	(0.041)	(0.022)	(0.025)
Constant	1.326	2.490	0.611
	(0.259)	(0.326)	(0.020)
Observations	550	550	550

Note: Standard errors are clustered at the major level. Column (1) reports the regression result on (1) in which β_1 captures the mean difference of major fit of SS students to that of LS students. Column (2) reports the regression result on (2) in which β_1 captures the mean difference of major fit of Economics students to that of non-Economics students. Column (3) is the full regression in (3).

Prestige premium means that Economics provides better chance of signaling by pooling with IM students, which is captured by SS students having higher additional value on Economics ($\hat{\gamma}_{Econ}$). In particular, the loss from the former is 1.645, whereas the loss from the latter amounts to 0.325; both are statistically significant at 1%.[2]

A.5 Regression of GPA on Demeaned Major Fit

As a robustness check of Table 5, we report the regression results of an alternative version of (6) in Table A.4 in which we replace $\lambda_{ij} = \lambda_{ij(i)}$ by its (student-specific) demeaned version, $\bar{\lambda}_{ij} := \lambda_{ij} - \frac{1}{J}\sum_{k=1}^{J} \lambda_{ik}$. We do not find significantly different results.

[2]See the explanation of (4) in Section 3.3.1 for how we interpret the coefficients in column (3).

Table A.4: GPA on Demeaned Major Fit for a Chosen Major

	(1) Total	(2) Lib Art	(3) Major-core
Major Fit (Demeaned)×Econ	0.036	-0.329	0.272
	(0.058)	(0.053)	(0.093)
Major Fit (Demeaned)×non-Econ	0.123	0.222	-0.215
	(0.061)	(0.097)	(0.211)
Regular	-0.093	-0.060	0.069
	(0.025)	(0.046)	(0.051)
Freshman GPA	0.324	0.276	0.437
	(0.030)	(0.025)	(0.022)
# Econ. in Freshman	-0.010	-0.030	0.062
	(0.005)	(0.004)	(0.007)
# Math. in Freshman	-0.000	-0.019	0.166
	(0.020)	(0.013)	(0.020)
# Lib. Art. in Freshman	-0.011	0.019	0.058
	(0.011)	(0.018)	(0.021)
# Business. in Freshman	0.007	0.011	0.027
	(0.015)	(0.019)	(0.022)
Observations	550	545	463

Note: Major specific intercepts ($\hat{\alpha}_j$) are omitted, and standard errors are clustered at the chosen major level.

B Supplementary Materials to Section 5

B.1 Distributional Consequences of Prestige Concerns

Let us assume that the unit mass of students is partitioned into two groups: "privileged" and "underprivileged" of mass m_P and m_U, respectively, where $m_P + m_U = 1$. The groups differ in their score distribution: for the privileged, v follows a CDF denoted by P while the distribution is U for the underprivileged group. We assume that P dominates U in the hazard rate order, i.e.,

$$\frac{1 - U(v)}{1 - P(v)}$$

decreases in v.[3] We recall that this implies that P first-order stochastically dominates U. In the next proposition, we argue that it is the disadvantaged group who particularly suffers from the distortion caused by the prestige concern.

Proposition 3. *As Δ and τ (weakly) increase,*

(i) the equilibrium share of the underprivileged in college A (resp. B) becomes lower (resp. higher). The equilibrium share of the unassigned underprivileged remains unchanged;

[3] This is weaker than assuming that P is greater than U in the likelihood ratio order.

(ii) the equilibrium utilitarian welfare of the underprivileged becomes lower if $\kappa_A q_A + \kappa_B q_B$ remains the same.

Proof. To prove Part (i), let us fix an equilibrium before an increase in Δ and τ. We start by showing that there is an equilibrium after the change of parameters under which the share of underprivileged assigned College A decreases. Recall from Theorem 2 that there exists an equilibrium after the change in which the prestige gap $\hat{\delta}$ and cutoff score $\hat{v}_A$ are weakly higher while $\hat{\alpha}$ is weakly lower. The share of underprivileged in college A is given by

$$\frac{m_U\,(1-U(\hat{v}_A))\,(1-G(\hat{\alpha}))}{[m_U\,(1-U(\hat{v}_A))+m_P\,(1-P(\hat{v}_A))]\,(1-G(\hat{\alpha}))}=\frac{m_U\,(1-U(\hat{v}_A))}{[m_U\,(1-U(\hat{v}_A))+m_P\,(1-P(\hat{v}_A))]}$$

when $\hat{v}_A$ is the cutoff score of college A. Since this cutoff score is weakly higher under at least one equilibrium after the change and since the above term is decreasing in $\hat{v}_A$ (by our assumption that P is greater than U in the hazard rate order), the share of underprivileged in college A falls for at least one equilibrium after the change.[4] We also need to show for any equilibrium after the change in parameters, there is an equilibrium before the change under which the share of underprivileged assigned College A is larger. The argument is the same as above and is thus omitted. This shows that the set of equilibrium shares of underprivileged in college A decreases.

Now, consider an arbitrary equilibrium. We know that the share of assigned underprivileged is

$$\frac{m_U\,(1-U(\underline{v}))}{m_U\,(1-U(\underline{v}))+m_P\,(1-P(\underline{v}))}$$

when $\underline{v}$ is the cutoff score for college B. Since $1-F(\underline{v})=\kappa_A+\kappa_B$, $\underline{v}$. Since this cutoff score does not depend on Δ and τ, the share of assigned underprivileged students is the same at any equilibrium. Note that this also implies that the set of equilibrium shares of underprivileged in college B increases. Hence, this proves Part (i).

We now move to the proof of Part (ii). Fix an equilibrium before an increase in Δ and τ. We start by showing that there is an equilibrium after the change under which the utilitarian welfare of the underprivileged group decreases (assuming $\kappa_A q_A + \kappa_B q_B$ remains the same). We consider the equilibrium after the change under which the share of underprivileged assigned College A decreases which exists by Part (i) of the proposition. Let T_j^1 and T_j^2 denote the sets of underprivileged student types assigned to major j before and after the parameter change, respectively. Then, $T_{AB} := T_A^1 \backslash T_A^2$ are the underprivileged student types whose assignment changes from A to B with the parameter change, while $T_{BA} := T_A^2 \backslash T_A^1$ are the underprivileged types whose assignment changes from B to A. Note that all other types do not change their

[4] It is easy to check that since $F = m_P P + m_U U$, P dominates F in the hazard rate order, which in turn, dominates U in the hazard rate order.

assignments going from the original equilibrium to the new one. Consider now a hypothetical situation in which all variables, both exogenous and endogenous, remain the same as in the original equilibrium while students are assigned as in the new equilibrium. Then, the utilities of students with types in $T\backslash(T_{AB} \cup T_{BA})$ do not change (since their assignments do not change). Next, students with types in T_{AB} and those with types in T_{BA} both get worse off since the former prefer A to B and the latter prefer B to A in the original equilibrium. Thus, the utilitarian welfare of underprivileged students becomes weakly lower in the hypothetical situation. Let us now fix the student assignment at that new equilibrium (i.e, that in the hypothetical situation) and change all the variables (i.e., τ, Δ, $\mathbb{E}_A[v]$ and $\mathbb{E}_B[v]$) from the original levels to the new ones. It is enough to show that the aggregate utility of underprivileged students from the major prestige decreases. This is what is stated in the following lemma.

In the sequel, we add a superscript 1 (resp., 2) for variables before (resp., after) the change. Further, we denote u_j^1 and p_j^1 (resp., u_j^2 and p_j^2) for the share of underprivileged and privileged students among students enrolled in major $j \in \{A, B, \varnothing\}$ at the equilibrium before (resp., after) the change.

Lemma 2. *The prestige part of the total welfare for underprivileged decreases after the change, i.e.,*

$$\sum_{j=A,B,\varnothing} \tau^1 \left[u_j^1 \kappa_j (\mathbb{E}_j^1[v] - \mathbb{E}[v])\right] \geqslant \sum_{j=A,B,\varnothing} \tau^2 \left[u_j^2 \kappa_j (\mathbb{E}_j^2[v] - \mathbb{E}[v])\right].$$

The argument for the lemma relies on the following claims.

Claim 2. *The share of underprivileged students before the change*[5] *is smaller in school A than in school B, i.e., $u_B^1 \geqslant u_A^1$.*

Proof. By definition, $u_B^1 \geqslant u_A^1$ is equivalent

$$\frac{m_U\left(1 - U(\underline{v}^1)\right) - m_U\left(1 - U(\hat{v}_A^1)\right)\left(1 - G(\hat{\alpha}^1)\right)}{\kappa_B} \geqslant \frac{m_U\left(1 - U(\hat{v}_A^1)\right)\left(1 - G(\hat{\alpha}^1)\right)}{\kappa_A}. \tag{4}$$

This can written as

$$1 - U(\underline{v}^1) \geqslant \frac{\kappa_A + \kappa_B}{\kappa_A}\left(1 - U(\hat{v}_A^1)\right)\left(1 - G(\hat{\alpha}^1)\right)$$

Now, using the market clearing conditions for college A and B, we obtain

$$1 - U(\underline{v}^1) \geqslant \frac{1 - F(\underline{v}^1)}{\left(1 - F(\hat{v}_A^1)\right)\left(1 - G(\hat{\alpha}^1)\right)}\left(1 - U(\hat{v}_A^1)\right)\left(1 - G(\hat{\alpha}^1)\right).$$

[5]The same argument shows that the same holds true after the change.

Hence, we only need to show that the inequality below holds

$$\frac{1-U(\underline{v}^1)}{1-F(\underline{v}^1)} \geqslant \frac{1-U(\hat{v}_A^1)}{1-F(\hat{v}_A^1)}.$$

This inequality holds because of the hazard rate dominance of F over U and the fact that $\underline{v}^1 \leqslant \hat{v}_A^1$. □

Claim 3. *We must have*

$$\sum_{j=A,B,\varnothing} u_j^1 \kappa_j(\mathbb{E}_j^1[v] - \mathbb{E}[v]) \geqslant \sum_{j=A,B,\varnothing} u_j^2 \kappa_j(\mathbb{E}_j^2[v] - \mathbb{E}[v]).$$

Proof. Recall that by Proposition 3-(i), $u_\varnothing^2 = u_\varnothing^1$. In addition, $\underline{v}^1 = \underline{v}^2$ and so $\mathbb{E}_\varnothing^1[v] = \mathbb{E}_\varnothing^2[v]$. Thus, the inequality in the statement of the claim holds if and only if

$$u_A^1 \kappa_A(\mathbb{E}_A^1[v] - \mathbb{E}[v]) + u_B^1 \kappa_B(\mathbb{E}_B^1[v] - \mathbb{E}[v]) \geqslant u_A^2 \kappa_A(\mathbb{E}_A^2[v] - \mathbb{E}[v]) + u_B^2 \kappa_B(\mathbb{E}_B^2[v] - \mathbb{E}[v])$$

which is equivalent to

$$\begin{aligned} &u_A^1 \kappa_A(\mathbb{E}_A^1[v] - \mathbb{E}[v]) + u_B^1 \kappa_B(\mathbb{E}_B^1[v] - \mathbb{E}[v]) \\ &\qquad \geqslant (u_A^1 + (u_A^2 - u_A^1))\kappa_A(\mathbb{E}_A^2[v] - \mathbb{E}[v]) + (u_B^1 + (u_B^2 - u_B^1))\kappa_B(\mathbb{E}_B^2[v] - \mathbb{E}[v]). \end{aligned}$$

Reorganizing the terms, this can be written as

$$\begin{aligned} &(u_A^1 - u_A^2)\kappa_A(\mathbb{E}_A^2[v] - \mathbb{E}[v]) + (u_B^1 - u_B^2)\kappa_B(\mathbb{E}_B^2[v] - \mathbb{E}[v]) \\ &\qquad \geqslant u_A^1 \kappa_A(\mathbb{E}_A^2[v] - \mathbb{E}_A^1[v_A]) + u_B^1 \kappa_B(\mathbb{E}_B^2[v] - \mathbb{E}_B^1[v]). \end{aligned} \tag{5}$$

Now, since for each $i = 1, 2$:

$$\kappa_A \mathbb{E}_A^i[v] + \kappa_B \mathbb{E}_B^i[v] + (1 - \kappa_A - \kappa_B)\mathbb{E}_\varnothing^i[v] = \mathbb{E}[v],$$

and, again, $\mathbb{E}_\varnothing^1[v] = \mathbb{E}_\varnothing^2[v]$, we know that

$$\kappa_A \left(\mathbb{E}_A^2[v] - \mathbb{E}_A^1[v]\right) = -\kappa_B \left(\mathbb{E}_B^2[v] - \mathbb{E}_B^1[v]\right). \tag{6}$$

Similarly, since for each $i = 1, 2$:

$$\kappa_A u_A^i + \kappa_B u_B^i + (1 - \kappa_A - \kappa_B) u_\varnothing^i = m_U,$$

and, again, $u_\varnothing^1 = u_\varnothing^2$, we know that

$$\kappa_A \left(u_A^2 - u_A^1\right) = -\kappa_B \left(u_B^2 - u_B^1\right). \tag{7}$$

Equations (6) and (7) above allow us to rewrite Equation (5) as follows

$$(u_A^1 - u_A^2)\kappa_A \left(\mathbb{E}_A^2[v] - \mathbb{E}_B^2[v]\right) \geqslant \left(u_A^1 - u_B^1\right) \kappa_A \left[\mathbb{E}_A^2[v] - \mathbb{E}_A^1[v]\right].$$

Note that the left-hand side is positive by part (i) of the proposition and the fact that $\mathbb{E}_A^2[v] \geqslant \mathbb{E}_B^2[v]$ while the right-hand side is negative by Claim 2 and the fact that $\mathbb{E}_A^2[v] \geqslant \mathbb{E}_A^1[v]$ which is proved in Theorem 2-(i) (i.e., $\hat{v}_A^2 \geqslant \hat{v}_A^1$). This completes the proof of the claim. □

Given Claim 3, in order to complete the proof of Lemma 2, it is enough to show that the prestige part of the welfare for underprivileged is nonpositive.

Claim 4. *For each $i = 1, 2$*

$$\sum_{j=A,B,\varnothing} u_j^i \kappa_j (\mathbb{E}_j^i[v] - \mathbb{E}[v]) \leqslant 0.$$

Proof. We claim that the distribution

$$(u_A^i \kappa_A / m_U, u_B^i \kappa_B / m_U, u_\varnothing^i (1 - \kappa_A - \kappa_B)/m_U)$$

is stochastically dominated by the distribution

$$(p_A^i \kappa_A / m_P, p_B^i \kappa_B / m_P, p_\varnothing^i (1 - \kappa_A - \kappa_B)/m_P).$$

This is enough for our purpose. Indeed, proceeding by contradiction, if

$$u_A^i \kappa_A (\mathbb{E}_A^i[v] - \mathbb{E}[v]) + u_B^i \kappa_B (\mathbb{E}_B^i[v] - \mathbb{E}[v]) + u_\varnothing^i (1 - \kappa_A - \kappa_B)(\mathbb{E}_\varnothing^i[v] - \mathbb{E}[v]) > 0$$

then

$$p_A^i \kappa_A (\mathbb{E}_A^i[v] - \mathbb{E}[v]) + p_B^i \kappa_B (\mathbb{E}_B^i[v] - \mathbb{E}[v]) + p_\varnothing^i (1 - \kappa_A - \kappa_B)(\mathbb{E}_\varnothing^i[v] - \mathbb{E}[v]) > 0$$

since $\mathbb{E}_A^i[v] - \mathbb{E}[v] \geqslant \mathbb{E}_B^i[v] - \mathbb{E}[v] \geqslant \mathbb{E}_\varnothing^i[v] - \mathbb{E}[v]$. But this would violate the zero sum nature of the aggregate utility from the major prestige.

In order to show the stochastic dominance property, we first need to show that

$$u_A^i \kappa_A / m_U \leqslant p_A^i \kappa_A / m_P.$$

Simple algebra shows that this is equivalent to

$$1 - U(\hat{v}_A^i) \leqslant 1 - P(\hat{v}_A^i)$$

which holds true given our assumption that P stochastically dominates U. Further, we have to show that

$$\left(u_A^i \kappa_A + u_B^i \kappa_B\right)/m_U \leqslant \left(p_A^i \kappa_A + p_A^i \kappa_B\right)/m_P$$

or equivalently,

$$u_\varnothing^i (1 - \kappa_A - \kappa_B)/m_U \geqslant p_\varnothing^i (1 - \kappa_A - \kappa_B)/m_P$$

Again, simple algebra shows that this is equivalent to

$$U(\underline{v}^i) \geqslant P(\underline{v}^i)$$

which, again, holds true given our assumption that P stochastically dominates U. □

We have shown that there is an equilibrium after the change in Δ and τ under which the utilitarian welfare of the underprivileged group decreases (assuming $\kappa_A q_A + \kappa_B q_B$ remains the same). We also need to show for any equilibrium after the change in parameters, there is an equilibrium before the change under which the utilitarian welfare of the underprivileged group is larger. The argument is the same as above and is thus omitted. This shows that the set of equilibrium utilitarian welfare of the underprivileged group decreases provided that $\kappa_A q_A + \kappa_B q_B$ remains the same. □

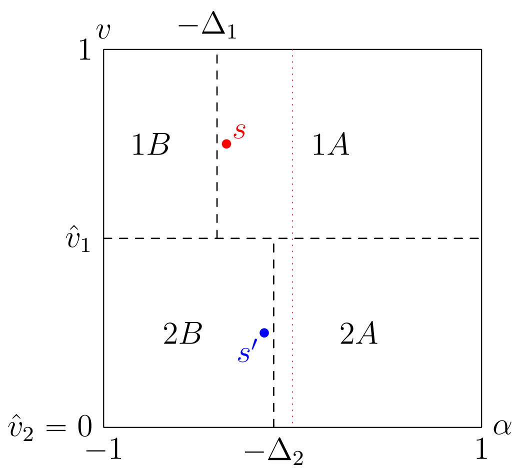
B.2 Immediate Major Choice versus Deferred Major Choice

Suppose that there are two colleges with equal capacity (=1/2), College 1 and College 2, and two majors, major A and major B, in each college. There is a department for each major j in college k, called Dept kj.

Let q_{kj} denote the quality of Dept kj. Assume that $\Delta_k := q_{kA} - q_{kB} > 0, k = 1, 2$ and $\Delta_j q := q_{1j} - q_{2j} > 0, j = A, B$: that is, major A in each college offers a higher quality than major B while college 1 offers a higher quality for each major than college 2. Letting ε_j denote the idiosyncratic preference for major $j = A, B$ as before, we assume there are no idiosyncratic preferences for colleges. This assumption is made to be consistent with our casual observation that the preference heterogeneity is likely smaller across colleges than across majors.

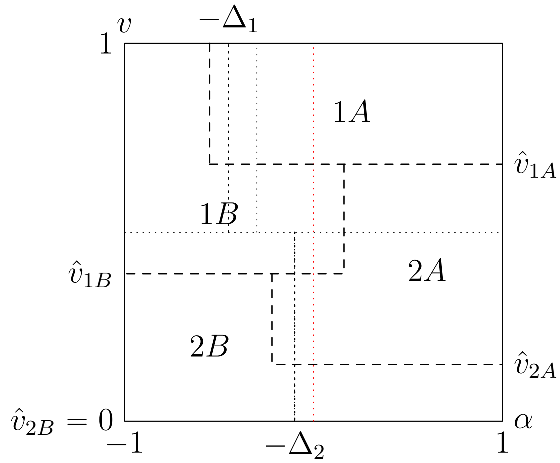
We consider two admission systems, college-based admission (CBA) and department-based admission (DBA). Under CBA, students get admitted to colleges and then freely choose their majors (or departments). Thus, there is no capacity constraint for each individual department, apart from the constraint imposed by the capacity of colleges. Under DBA, students get admitted to departments under the constraint that Dept kj cannot enrol more than its fixed capacity given *exogenously* as κ_{kj}.

In an equilibrium of CBA, students are assigned as in the following figure:



That the threshold α is equal to $-\Delta_k$ in each college $k = 1, 2$ means that within each college, there is no distortion due to the prestige gap between the two majors. In other words, only college-specific prestige exists with college 1 being more prestigious than college 2, which is a source of distortion under CBA. For instance, consider two student types s and s' in the above figure, whose idiosyncratic preferences are $(\varepsilon_A, \varepsilon_B)$ and $(\varepsilon'_A, \varepsilon'_B)$, respectively. Let $\alpha = \varepsilon_A - \varepsilon_B$ and $\alpha' = \varepsilon'_A - \varepsilon'_B$ and note that $\alpha' > \alpha$. If we move s from Dept $1A$ to Dept $2B$ and s' from Dept $2B$ to Dept $1A$, then the utilitarian welfare will change by $\varepsilon_B + \varepsilon'_A - (\varepsilon_A + \varepsilon'_B) = (\varepsilon'_A - \varepsilon'_B) - (\varepsilon_A - \varepsilon_B) = \alpha' - \alpha > 0$.

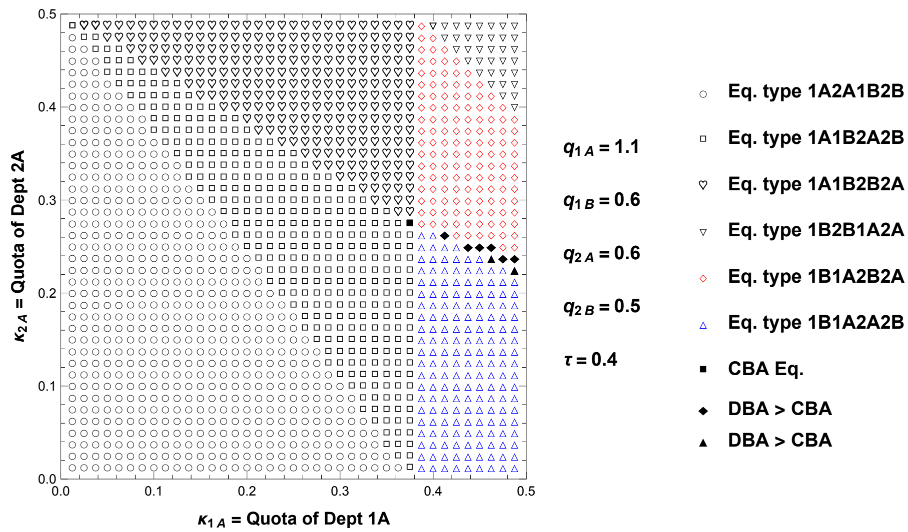
Under DBA, however, the prestige gap can exist between different departments within the same college as well as between different colleges. The following figure illustrates one equilibrium assignment under DBA (with certain parametric specifications):



As before, college 1 is more prestigious than college 2: the cutoff scores are uniformly higher in college 1 than in college 2. Differently from CBA, however, there is also *within-college prestige gap*, i.e., major A is more prestigious than major B in each college, which is another source of distortion. This leads us to expect that CBA may well perform better than DBA in terms of students' welfare.

To compare the two systems, we have performed a numerical analysis of DBA with different parametric specifications as in the figure below, where each equilibrium type under DBA is labeled according to the descending order of cutoff scores.[6]

[6]For instance, "Eq. type 1A2A1B2B" means $\hat{v}_{1A} > \hat{v}_{2A} > \hat{v}_{1B} > \hat{v}_{2B}$. Our numerical analysis shows that the equilibrium is unique under each parametric specification.



Notice just a few of filled diamonds and filled triangles that correspond to the cases in which the student welfare is higher under DBA than under CBA. In all other cases, CBA performs better than DBA, as was expected.[7]

B.3 Signal Accuracy

As mentioned in Section 5, less accurate signals can reduce the possibility for majors to screen students in terms of their true ability. Hence, less accurate signals may reduce the prestige gap and, so, be welfare improving. We make this point formal in this section and make explicit what type of signal coarsening will eventually allow to increase total welfare.

How accurately the score or signal v reflects the student ability θ can be captured by the "variability" of the conditional expectation $\mathbb{E}[\theta\,|v]$ with more accurate signal corresponding to greater variability in a sense to be made precise. In our setup, where the signal is unbiased, i.e., $\mathbb{E}[\theta\,|v] = v$, this reduces to the variability of the signal v. Hence, we will simply refer to a signal as a cumulative distribution function (CDF, hereafter) for v and order signals based on the variability/precision of the CDFs. In the sequel, we restrict our attention to CDFs that are continuous and strictly increasing on their supports. Finally, given a CDF F_i and measurable set S, we will let $\mathbb{E}_i[\theta\,|v \in S]$ be the expectation of ability θ given that the score v belongs to S when v is distributed according to F_i.

Notions of signal precision We consider the following order to compare signals in terms of their precision. Suppose that there are two signals with distributions F_1 and F_2, whose supports

[7]CBA and DBA are equivalent in terms of the student assignment and welfare in a single case of filled square.

are $[\underline{v}_1, \overline{v}_1]$ and $[\underline{v}_2, \overline{v}_2]$, respectively.[8] We say that signal F_1 is **more supermodular precise** than signal F_2 if for $1 \geqslant c' \geqslant c \geqslant 0$:

$$F_1^{-1}(c') - F_1^{-1}(c) \geqslant F_2^{-1}(c') - F_2^{-1}(c)$$

which in our setup with unbiased signals is equivalent to

$$\mathbb{E}_1\left[\theta \,\middle|\, v = F_1^{-1}(c')\right] - \mathbb{E}_1\left[\theta \,\middle|\, v = F_1^{-1}(c)\right] \geqslant \mathbb{E}_2\left[\theta \,\middle|\, v = F_2^{-1}(c')\right] - \mathbb{E}_2\left[\theta \,\middle|\, v = F_2^{-1}(c)\right]. \quad (8)$$

In words, the more supermodular precise signal, F_1, has a (normalized) conditional expectation function that is more sensitive to changes in c than the less sensitive F_2 at every c.[9]

Welfare implication of signal precision We now explain how and under what conditions less accurate signals can reduce the prestige gap and, eventually, be welfare improving.

In the sequel, to perform comparative statics comparing sets of equilibria that arise from parameter changes, we use the notion of *weak-set order* following Che, Kim and Kojima (2021) and introduced in Section 2.2.

Proposition 4. *Assume we switch from signal F_1 to signal F_2 where F_1 is more supermodular precise than F_2,*

(i) the equilibrium prestige gap becomes lower;

(ii) the equilibrium utilitarian welfare becomes higher.

Before we move to the proof of the above result, let us provide an intuition for the result. First, it is easily shown that if F_1 is more supermodular precise than F_2 then for $c' \geqslant c$,

$$\mathbb{E}_1\left[\theta \,\middle|\, v \geqslant F_1^{-1}(c')\right] - \mathbb{E}_1\left[\theta \,\middle|\, v \geqslant F_1^{-1}(c)\right] \geqslant \mathbb{E}_2\left[\theta \,\middle|\, v \geqslant F_2^{-1}(c')\right] - \mathbb{E}_2\left[\theta \,\middle|\, v \geqslant F_2^{-1}(c)\right] \quad (9)$$

where we simply replaced the equalities in (8) by inequalities. Now, consider the equilibrium prestige gap δ_1 under distribution F_1. The cutoff score for major B must be $F_1^{-1}(c)$ for $c = \kappa_A + \kappa_B$ while for major A, it must be $F_1^{-1}(c')$ where $c' \geqslant c$.[10] Hence, the equilibrium prestige gap δ_1 corresponds to the left-hand side of (9) for these specific c' and c.

Now consider signal distribution F_2 which is less supermodular precise than F_1. Assume that all agents believe that the prestige gap is given by δ_1. One can compute the new prestige gap where students' decisions remain unchanged but where majors adjust their (market-clearing)

[8] In this section, we will allow the support of the CDF of the signal distribution to vary. In particular, the support may not be $[0, 1]$ anymore. All our results in Section 2 extend to this context in a straightforward way.

[9] In Shaked and Shanthikumar (2007, Theorem 3.B.14), it is shown that if X and Y are random variables having the same finite support, then X is more supermodular precise than Y if and only if X and Y have the same distributions. This motivates our modelling choice in the current section to allow the support of the CDFs for signal distributions to vary.

[10] It is easily checked that to ensure market-clearing, c' must be equal to $1 - \kappa_A(1 - G(\hat{\alpha}(\delta_1)))$.

cutoff scores to the new signal distribution. This new prestige gap now corresponds to the right-hand side of (9) for the c' and c as specified above. Hence, by definition of supermodular precision, the new prestige gap is smaller than δ_1: the average score of students enrolled in major A decreases more than the average score of students enrolled in B. Now, if we let agents reoptimize, students' demand for major A decreases which makes A even less selective and so makes the resulting equilibrium prestige gap smaller. This is the intuition behind the proof of Proposition 4-(i). As for Part (ii), this simply comes from our previous observation that a smaller prestige gap between majors incentivizes students to take more into account their major fits in their major applications and is thus welfare-improving.[11]

Proof of Proposition 4. Assume first that F_1 is more supermodular precise than F_2. We start by proving Part (i). For this purpose, we first state and prove the following lemma.

Lemma 3. *Signal F_1 is more supermodular precise than signal F_2 if for $c' \geqslant c$:*

$$\mathbb{E}_1\left[\theta \left| v \geqslant F_1^{-1}(c')\right.\right] - \mathbb{E}_1\left[\theta \left| v \geqslant F_1^{-1}(c)\right.\right] \geqslant \mathbb{E}_2\left[\theta \left| v \geqslant F_2^{-1}(c')\right.\right] - \mathbb{E}_2\left[\theta \left| v \geqslant F_2^{-1}(c)\right.\right].$$

Proof. Since F_1 is more supermodular precise than F_2, by definition, $\mathbb{E}_1\left[\theta \left| F_1(v) = \hat{c}\right.\right] - \mathbb{E}_2\left[\theta \left| F_2(v) = \hat{c}\right.\right]$ is nondecreasing in $\hat{c}$. Hence, given that $c' \geqslant c$, the uniform distribution over $[c', 1]$ stochastically dominates the uniform distribution over $[c, 1]$. We obtain

$$\int_{\hat{c} \geqslant c'} \frac{1}{1-c'} \left[\mathbb{E}_1\left[\theta \left| F_1(v) = \hat{c}\right.\right] - \mathbb{E}_2\left[\theta \left| F_2(v) = \hat{c}\right.\right]\right] d\hat{c}$$
$$\geqslant \int_{\hat{c} \geqslant c} \frac{1}{1-c} \left[\mathbb{E}_1\left[\theta \left| F_1(v) = \hat{c}\right.\right] - \mathbb{E}_2\left[\theta \left| F_2(v) = \hat{c}\right.\right]\right] d\hat{c}. \tag{10}$$

Now, by standard arguments, $F_1(v)$ and $F_2(v)$ – where $v \sim F_1$ and $v \sim F_2$ respectively – are both uniform distributions over $[0, 1]$.[12] So, for any $c \in [0, 1]$,

$$\mathbb{E}_i\left[\theta \left| F_i(v) \geqslant c\right.\right] = \int_{\hat{c} \geqslant c} \frac{1}{1-c} \mathbb{E}_i\left[\theta \left| F_i(v) = \hat{c}\right.\right] d\hat{c} \tag{11}$$

for each $i = 1, 2$. Thus, combining (10) and (11), we have

$$\mathbb{E}_1\left[\theta \left| F_1(v) \geqslant c'\right.\right] - \mathbb{E}_2\left[\theta \left| F_2(v) \geqslant c'\right.\right] \geqslant \mathbb{E}_1\left[\theta \left| F_1(v) \geqslant c\right.\right] - \mathbb{E}_2\left[\theta \left| F_2(v) \geqslant c\right.\right]$$

[11] One can imagine many possible orders to compare signal precision across distributions. For instance, one may use the standard (and weaker) notion of mean preserving spread. As it turns out, such a notion is not strong enough to guarantee that Proposition 4 holds under this weaker order. The reason is as follows: if one switches the signal distribution from F_1 to F_2 where F_1 is a mean preserving spread of F_2, even though the average score of students enrolled in major A decreases, this does not necessarily imply that the average score of students enrolled in major B increases, it may actually decrease. Further, this decrease can be strong enough that the prestige gap eventually increases.

[12] This is sometimes referred to as the probability integral transform Theorem. Note that this holds since we are restricting ourselves to CDFs that are continuous and strictly increasing.

which yields the desired result. □

Now, let us fix an equilibrium δ_1 when the signal is F_1. We start by showing that there is an equilibrium δ_2 when the signal is F_2 satisfying $\delta_2 \leqslant \delta_1$. Let us denote ϕ_i the mapping defined in (13) when the signal v is distributed according to F_i for $i = 1, 2$. We claim that $\phi_2(\delta_1) \leqslant \phi_1(\delta_1)$. Since, by definition, $\phi_1(\delta_1) = \delta_1$, this will imply that $\phi_2(\delta_1) \leqslant \delta_1$. This, together with Theorem 1, yields that the restriction of ϕ_2 to $[0, \delta_1]$ is a nondecreasing self-map. Hence, ϕ_2 has a fixed point weakly smaller than δ_1.

Now, to see that $\phi_2(\delta_1) \leqslant \phi_1(\delta_1)$, set $\hat{\alpha}(\delta_1) = \max\{-\Delta - \tau\delta_1, -1\}$ and $c := 1 - \kappa_A \backslash (1 - G(\hat{\alpha}(\delta_1)))$. Further denote $\hat{v}_{A,1}(\delta_1) = F_1^{-1}(c)$ as well as $\hat{v}_{A,2}(\delta_1) = F_2^{-1}(c)$. Similarly, let us set $\underline{v}_1 = F_1^{-1}(1 - \kappa_A - \kappa_B)$ and $\underline{v}_2 = F_2^{-1}(1 - \kappa_A - \kappa_B)$.

We recall that

$$\phi_1(\delta_1) = \frac{\kappa_A + \kappa_B}{\kappa_B}(e_1(\hat{v}_{A,1}(\delta)) - e_1(\underline{v}_1))$$

while

$$\phi_2(\delta_1) = \frac{\kappa_A + \kappa_B}{\kappa_B}(e_2(\hat{v}_{A,2}(\delta)) - e_2(\underline{v}_2))$$

where for any $\hat{v}$, $e_i(\hat{v}) = \mathbb{E}_i\left[\theta \,\middle|\, v \geqslant \hat{v}\right]$. So, in order to show that $\phi_2(\delta_1) \leqslant \phi_1(\delta_1)$, we need to show that

$$e_1(\hat{v}_{A,1}(\delta_1)) - e_1(\underline{v}_1) \geqslant e_2(\hat{v}_{A,2}(\delta_1)) - e_2(\underline{v}_2)$$

which is equivalent to

$$\begin{aligned}\mathbb{E}_1\left[\theta \,\middle|\, v \geqslant F_1^{-1}(c)\right] &- \mathbb{E}_1\left[\theta \,\middle|\, v \geqslant F_1^{-1}(1 - \kappa_A - \kappa_B)\right] \\ &\geqslant \mathbb{E}_2\left[\theta \,\middle|\, v \geqslant F_2^{-1}(c)\right] - \mathbb{E}_2\left[\theta \,\middle|\, v \geqslant F_2^{-1}(1 - \kappa_A - \kappa_B)\right]. \end{aligned} \tag{12}$$

Finally, by definition of an equilibrium, $F_1^{-1}(c) = \hat{v}_{A,1}(\delta_1) \geqslant \underline{v}_1 = F_1^{-1}(1 - \kappa_A - \kappa_B)$, and given that inverse distribution functions are nondecreasing, we must have $c \geqslant 1 - \kappa_A - \kappa_B$. Hence, the above is implied by the characterization provided in Lemma 3 and our assumption that F_1 is more supermodular precise than F_2. So we proved that there is an equilibrium δ_2 when the signal is F_2 satisfying $\delta_2 \leqslant \delta_1$. To complete the proof of Part (i), we also need to show for any equilibrium δ_2 when the signal is F_2, there is an equilibrium δ_1 when the signal is F_1 satisfying $\delta_1 \geqslant \delta_2$. The argument is the same as above and is thus omitted.

Now, we move to the proof of Part (ii). Fix $i = 1, 2$ and let δ_i be equilibrium prestige gaps when $v \sim F_i$ so that $\delta_2 \leqslant \delta_1$ which is well-defined as proved in (i). Now, we consider an economy where agents receive signal $F_i(v)$, which, as we already mentioned, is distributed according to $U[0, 1]$. In this economy, college A admits students with $c \geqslant c_i$ where $c_i := 1 - \kappa_A \backslash (1 - G(\hat{\alpha}(\delta_i)))$ while college B admits students with $c \geqslant \underline{c}$ where $\underline{c} := 1 - \kappa_A - \kappa_B$. Given a realization of the signal $c \sim U[0,1]$, the conditional expectation of the student's ability $\mathbb{E}\left[\theta \,|c \geqslant c_i\right]$ is simply defined as $\mathbb{E}_i\left[\theta \,\middle|\, v \geqslant F_i^{-1}(c_i)\right]$. The total welfare in this economy where signal is distributed according to $U[0,1]$ is, by construction, the same as the one in the original economy where signal is $v \sim F_i$. Hence, we will compare welfare of these economies, say $i = 1, 2$, where the signal is uniform over $[0,1]$. Note that, since $\delta_2 \leqslant \delta_1$, we must have $c_2 \leqslant c_1$. The remaining part of the proof is similar to that of Theorem 2-(ii).

Indeed, let T_j^1 and T_j^2 denote the sets of student types assigned to major j in economy 1 and 2, respectively. Then, $T_{AB} := T_A^2 \backslash T_A^1$ are the student types whose assignment changes from A to B when switching from economy 2 to economy 1, while $T_{BA} := T_A^1 \backslash T_A^2$ are the types whose assignment changes from B to A. Note that all other types do not change their assignments going from the economy 2 to economy 1. Consider now a hypothetical situation in which all variables (including the prestige gap) remain the same as in economy 1 while students are assigned as in economy 2. Then, the utilities of students with types in $T \backslash (T_{AB} \cup T_{BA})$ do not change (since their assignments do not change). Next, students with types in T_{AB} and those with types in T_{BA} both get worse off since the former prefer A to B and the latter prefer B to A in economy 1. Thus, the utilitarian welfare becomes weakly lower in the hypothetical situation. Let us now fix the student assignment and change all the variables from economy 2 to economy 1. As a consequence, the aggregate utility from the major quality becomes weakly lower since the aggregate utility from the major prestige does not change due to its zero sum nature, as argued via (2). □

B.4 Restricting Application

As we discussed in Section 5, the design of application or admission system is another important element that affects the way student's major choice and their prestige concern interact. We have so far focused on the system of *unrestricted application* (or *UA*) which allows students to apply to both majors. In reality, however, students may only have limited opportunities to apply to different majors due to the design of college admission system and/or to application costs. We capture this situation via what we call *restricted application* (or *RA*), where each student is only allowed to apply to one major. Under UA, each student's preference over different majors is affected by the cutoff scores only through their effect on the prestige of the majors. Under RA, however, the admission chance in each major associated with its cutoff score is another channel via which the cutoff scores affect the major preference of students who must decide which major to apply to. A key trade-off here is that a more prestigious major carries a higher risk of admission failure due to its higher cutoff score. This risk will make a more prestigious major less attractive to students and help alleviate the distortionary effect of prestige concern. In this section, we analyze a model of RA based on the baseline model, describe the equilibrium assignment of students, and draw welfare implications.

Setup We adopt the same setup as in Section 2.1 with some modifications. First, we assume that the major fits of each student type are single-dimensional in the following sense: for $\alpha \in [-1, 1]$, her fit for major $j = A, B$ is $\varepsilon_j(\alpha) \in [0, 1]$ with $\varepsilon_A(\alpha)$ (resp., $\varepsilon_B(\alpha)$) continuously increasing (resp., decreasing) in α.[13]

Next, we assume that each student cannot observe her own score and only knows its distribution given by the cdf F. The reason behind this assumption is twofold. First, it is practical in that the college applicants in reality face some uncertainty about their chances of admission, which is often due to uncertainty about how they are evaluated. The assumption that every student faces the same uncertainty (represented by the cdf F), though rather restrictive, enables us to get a clean comparison between the two admission systems due to the aforementioned admission risk. Second, in our continuum agent model with no aggregate uncertainty, the equilibrium cutoff scores are deterministic, which means that if students were to know their scores, then they could perfectly predict their admission outcomes, so RA and UA would yield the same equilibrium outcome.

B.4.1 Analysis of restricted application system

Let us denote each equilibrium variable under RA by adding a superscript r to the same variable under UA: for instance, $\hat{v}_j^r$ denotes the cutoff score for major $j = A, B$ while $\mathbb{E}_j^r[v]$ denotes the average score of students enrolled in $j = A, B, \varnothing$. Given any cutoff score $\hat{v}_j^r \in [0, 1]$ for major $j = A, B$, the expected payoff of type-α student from applying to j is given as

$$\begin{aligned} u_j^r(\alpha; \hat{v}_A^r, \hat{v}_B^r) := (1 - F(\hat{v}_j^r))\big(\varepsilon_j(\alpha) + q_j + \tau(\mathbb{E}_j^r[v] - \mathbb{E}[v])\big) \\ + F(\hat{v}_j^r)\tau(\mathbb{E}_\varnothing^r[v] - \mathbb{E}[v]). \end{aligned} \tag{13}$$

The first term is the payoff from getting admitted to j with probability $1 - F(\hat{v}_j^r)$ while the second term is the payoff from failing to get in j with probability $F(\hat{v}_j^r)$. Our assumption here is that the student ends up in the null major in this event (even though some majors might have vacant seats as a result).[14] Observe that without knowing her score, each student's application

[13] Our setup in Section 2 assumes that each student type $t = (\varepsilon_A, \varepsilon_B, v) \in [0, 1]^3$ is drawn according to some distribution and that this induces random variable $\alpha := \varepsilon_A - \varepsilon_B$ with cdf G. Our assumption here implicitly puts restrictions on the distribution of types: given $\alpha \in [-1, 1]$, there is a unique pair $(\varepsilon_A(\alpha), \varepsilon_B(\alpha))$ with positive density for which $\alpha = \varepsilon_A(\alpha) - \varepsilon_B(\alpha)$ and, in addition, this pair is given in such a way that $\varepsilon_A(\alpha)$ and $\varepsilon_B(\alpha)$ are increasing and decreasing in α, respectively.

[14] An alternative assumption would be that those students who have failed to get in the college they applied to could participate in a second round to get the available vacant seats in the unfilled majors. Of course, this

decision depends only on her α. Since utilities u_A^r and u_B^r are increasing and decreasing in α, respectively, any equilibrium must involve some threshold $\hat{\alpha}^r \in [-1, 1]$ such that each student applies to A (resp., B) if $\alpha > \hat{\alpha}^r$ (resp., $\alpha < \hat{\alpha}^r$). Letting T_j^r denote the set of types enrolling in major $j = A, B, \varnothing$, we have

$$T_A^r = \{(\alpha, v) \mid \alpha \geqslant \hat{\alpha}^r \text{ and } v \geqslant \hat{v}_A^r\} \tag{14}$$

$$T_B^r = \{(\alpha, v) \mid \alpha < \hat{\alpha}^r \text{ and } v \geqslant \hat{v}_B^r\} \tag{15}$$

(see the areas in Figure B.3 that are enclosed by the thick solid lines). So

$$\mathbb{E}_j^r[v] = \frac{\int_{\hat{v}_j^r}^1 v dF(v)}{1 - F(\hat{v}_j^r)} = e(\hat{v}_j^r) \text{ for each } j = A, B.$$

Then, $T_\varnothing^r = T \backslash (T_A^r \cup T_B^r)$ while $\mathbb{E}_\varnothing^r[v]$ can be obtained from

$$\sum_{j=A,B,\varnothing} \kappa_j \mathbb{E}_j^r[v] = \mathbb{E}[v]. \tag{16}$$

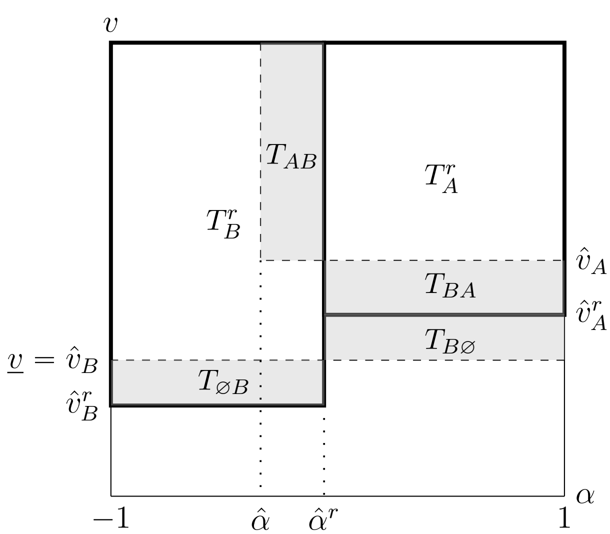


Figure B.3: Student Assignment under RA and Its Comparison with UA

The equilibrium assignment under RA is then determined by a tuple $(\hat{\alpha}^r, \hat{v}_A^r, \hat{v}_B^r)$ that satisfies the following conditions: the capacity constraints for major A and B given as

$$(1 - G(\hat{\alpha}^r))(1 - F(\hat{v}_A^r)) \leqslant \kappa_A \text{ (with equality if } \hat{v}_A^r > 0) \tag{17}$$

$$G(\hat{\alpha}^r)(1 - F(\hat{v}_B^r)) \leqslant \kappa_B \text{ (with equality if } \hat{v}_B^r > 0), \tag{18}$$

does not create any difference if all seats are allocated in the first round. We provide conditions in Lemma 4 below under which this occurs.

respectively; and the incentive constraint for the threshold type $\hat{\alpha}^r$ given as

$$u_A^r(\hat{\alpha}^r; \hat{v}_A^r, \hat{v}_B^r) \geqslant (\leqslant) u_B^r(\hat{\alpha}^r; \hat{v}_A^r, \hat{v}_B^r) \text{ if } \hat{\alpha}^r < 1 \text{ (if } \hat{\alpha}^r > -1). \tag{19}$$

Note that for an interior $\hat{\alpha}^r \in (-1, 1)$, this condition requires $u_A^r(\hat{\alpha}^r; \hat{v}_A^r, \hat{v}_B^r) = u_B^r(\hat{\alpha}^r; \hat{v}_A^r, \hat{v}_B^r)$. Note also that $u_A^r(\alpha; \hat{v}_A^r, \hat{v}_B^r) < (>) u_B^r(\alpha; \hat{v}_A^r, \hat{v}_B^r)$ if $\alpha < (>) \hat{\alpha}^r$.

We now show that relative to UA, RA makes major A less competitive (i.e., lowers its cutoff score), alleviating the distortionary effect of prestige concern and enhancing the student welfare if major B is fully enrolled in equilibrium (i.e., $\hat{v}_B^r > 0$).[15]

Proposition 5. *Suppose that the application system switches from UA to RA. In equilibria with nonnegative prestige gaps (i.e., $\hat{\delta}, \hat{\delta}^r \geqslant 0$),*

(i) the cutoff score for A becomes lower while the threshold type $\hat{\alpha}^r$ becomes higher[16]*;*

(ii) the equilibrium utilitarian welfare becomes higher, when restricted to equilibria under RA with $\hat{v}_B^r > 0$.

Before moving to the proof of this result, let us comment on its implications. The second part of Part (i)—that is, the higher preference cutoff under RA—implies that the types applying, and assigned, to A under RA have better fits for major A, relative to those assigned to A under UA, which reflects the fact that A becomes less attractive under RA due to its high risk of admission failure so that only those with tighter fits apply to A. To see the welfare effect of this change, see Figure B.3 in which each shaded area $T_{jj'}$ corresponds to the set of types who are assigned to j under UA and j' under RA. With the increase of the threshold type from $\hat{\alpha}$ to $\hat{\alpha}^r$, the types in T_{AB} assigned to A under UA are being replaced by the types in T_{BA} under RA who have better fits for major A. There is another benefit from switching to RA: the types in $T_{B\varnothing}$ assigned to B under UA are being replaced by those in $T_{\varnothing B}$ under RA who have better fits for major B. This is due to the nature of RA by which the types in $T_{B\varnothing}$ who apply to A without success have no chance to get in B under RA, which enables the types in $T_{\varnothing B}$ with lower scores but better fits for major B to get in B.[17]

Proof of Proposition 5. Proof of Part (i) Let us first fix an equilibrium with some $\hat{\alpha}$, $\hat{v}_A$, and $\hat{\delta} \geqslant 0$ under UA. We show that there exists an equilibrium with $\hat{\alpha}^r \geqslant \hat{\alpha}$ and $\hat{\delta}^r \geqslant 0$ under RA.

[15] Lemma 4 below provides a condition that guarantees $\hat{v}_B^r > 0$ in equilibrium.

[16] It is ambiguous whether the prestige gap also becomes lower under RA since $\hat{v}_B^r$ (as well as $\hat{v}_A^r$) becomes lower under RA.

[17] One potential drawback of RA is that it may entail some vacant seats in major B—so $\hat{v}_B^r = 0$—particularly if A is so popular (due to a significant quality gap, for instance) that B does not draw enough applicants to fill its seats. In this case, the student welfare can fall below that under UA, as shown in Appendix B.4.3

To do so, assume $\hat{\alpha} > -1$ (since otherwise there is nothing to prove). Define $\alpha_0 \in [-1, 1]$ such that $1 - G(\alpha_0) = \frac{\kappa_A}{\kappa_A + \kappa_B}$. By our assumption that $1 - G(-\Delta) \geqslant \frac{\kappa_A}{\kappa_A + \kappa_B}$, we have $\alpha_0 \geqslant -\Delta$. Thus, we also have $\hat{\alpha} \leqslant -\Delta \leqslant \alpha_0$ since $\hat{\delta} \geqslant 0$ implies $\hat{\alpha} = \max\{-\Delta - \tau\hat{\delta}, -1\} = -\Delta - \tau\hat{\delta} \leqslant -\Delta$.

Let us construct a mapping ψ which maps each $\alpha \in [-1, 1]$ to some $\alpha' \in [-1, 1]$ and whose fixed point will correspond to an equilibrium under RA. First, fix any $\alpha \in [-1, 1]$ and let $v_A^r(\alpha)$ be the score $v \in [0, 1]$ satisfying

$$(1 - G(\alpha))(1 - F(v)) = \kappa_A. \tag{20}$$

If the LHS of this equation is smaller than the RHS for every $v \in [0, 1]$, then let $v_A^r(\alpha) = 0$. Note that $v_A^r(\hat{\alpha}) = \hat{v}_A$ and $v_A^r(\alpha_0) = \underline{v}$. We define $v_B^r(\alpha)$ analogously by replacing $1 - G(\alpha)$ in (20) with $G(\alpha)$. Given the tuple $(\alpha, v_A^r(\alpha), v_B^r(\alpha))$, let us consider an assignment in which each student type $(\tilde{\alpha}, \tilde{v})$ is assigned to A (B, resp.) if $\tilde{\alpha} \geqslant \alpha$ ($\tilde{\alpha} < \alpha$, resp.) and $\tilde{v} \geqslant v_A^r(\alpha)$ ($\tilde{v} \geqslant v_B^r(\alpha)$, resp.) while all other types are assigned to $\varnothing$. Let T_j^r denote a set of types assigned to $j = A, B, \varnothing$ under this assignment and $\mathbb{E}_j^r[v]$ denote their average score. Then, $\mathbb{E}_j^r[v] = e(v_j^r(\alpha))$ for $j = A, B$ while $\mathbb{E}_\varnothing^r[v]$ then follows from (16). Substituting these into (13), we define $\psi(\alpha)$ to be a unique $\alpha' \in [-1, 1]$ satisfying

$$u_A^r(\alpha'; v_A^r(\alpha), v_B^r(\alpha)) = u_B^r(\alpha'; v_A^r(\alpha), v_B^r(\alpha)),$$

unless the LHS of this equation is greater (smaller, resp.) than the RHS for every $\alpha' \in [-1, 1]$, in which case we let $\psi(\alpha) = -1$ ($\psi(\alpha) = 1$, resp.). It is straightforward to see that if $\hat{\alpha}^r$ is a fixed point of ψ, then the threshold type $\hat{\alpha}^r$ together with the cutoff scores $\hat{v}_j^r = v_j^r(\hat{\alpha}^r), j = A, B$ can constitute an equilibrium assignment under RA.

We will show that $\psi(\hat{\alpha}) \geqslant \hat{\alpha}$ and $\psi(\alpha_0) \leqslant \alpha_0$, which will imply that there exists some $\alpha^r \in [\hat{\alpha}, \alpha_0]$ such that $\psi(\alpha^r) = \alpha^r$, as desired. To first prove $\psi(\alpha_0) \leqslant \alpha_0$, note that since $1 - G(\alpha_0) = \frac{\kappa_A}{\kappa_A + \kappa_B}$, (20) implies $v_A^r(\alpha_0) = \underline{v}$. Likewise, $v_B^r(\alpha) = \underline{v}$. This in turn implies that $\mathbb{E}_A^r[v] = \mathbb{E}_B^r[v]$. Using these observations, we have

$$\begin{aligned}
&u_A^r(\alpha'; v_A^r(\alpha_0), v_B^r(\alpha_0)) - u_B^r(\alpha'; v_A^r(\alpha_0), v_B^r(\alpha_0)) \\
&= (1 - F(\underline{v}))\big(\varepsilon_A(\alpha') + q_A - \varepsilon_B(\alpha') - q_B\big) = (1 - F(\underline{v}))\big(\alpha' + \Delta\big) = 0,
\end{aligned}$$

so $\alpha' = \psi(\alpha_0) = -\Delta \leqslant \alpha_0$.

To prove $\psi(\hat{\alpha}) \geqslant \hat{\alpha}$, let T_j denote the set of types who enroll in $j = A, B, \varnothing$ under UA. We next prove a couple of claims:

Claim 5. $v_B^r(\hat{\alpha}) \leqslant \hat{v}_B = \underline{v} \leqslant \hat{v}_A = v_A^r(\hat{\alpha})$

Proof. By comparing (10) and (20), we have $\hat{v}_A = v_A^r(\hat{\alpha})$. To show that $v_B^r(\hat{\alpha}) \leqslant \hat{v}_B = \underline{v}$, observe that

$$T' := \{(\tilde{\alpha}, \tilde{v}) : \tilde{\alpha} < \hat{\alpha} \text{ and } \tilde{v} \geqslant \hat{v}_B = \underline{v}\} \subset T_B. \tag{21}$$

First, if the measure of T_B^r falls short of κ_B, then we have $v_B^r(\hat{\alpha}) = 0 \leqslant \hat{v}_B$ by definition of $v_B^r(\cdot)$. If the measure of T_B^r is equal to κ_B and thus equal to the mass of T_B, then comparing T' in (21) and T_B^r in (15) yields $v_B^r(\hat{\alpha}) \leqslant \underline{v}$. □

Claim 6.

$$\big(F(v_A^r(\hat{\alpha})) - F(v_B^r(\hat{\alpha}))\big)\mathbb{E}_\varnothing^r[v] \leqslant \int_{v_B^r(\hat{\alpha})}^{v_A^r(\hat{\alpha})} \tilde{v} dF(\tilde{v}). \tag{22}$$

Proof. Observe first that

$$T_\varnothing^r = \{(\tilde{\alpha}, \tilde{v}) : \text{either } \tilde{\alpha} \geqslant \hat{\alpha} \text{ and } \tilde{v} < v_A(\hat{\alpha}) \text{ or } \tilde{\alpha} < \hat{\alpha} \text{ and } \tilde{v} < v_B^r(\hat{\alpha})\},$$

which can be partitioned into two sets as follows:

$$T_{\varnothing+}^r = \{(\tilde{\alpha}, \tilde{v}) : \tilde{\alpha} \geqslant \hat{\alpha} \text{ and } v_B^r(\hat{\alpha}) \leqslant \tilde{v} < v_A^r(\hat{\alpha})\} \tag{23}$$

$$T_{\varnothing-}^r = \{(\tilde{\alpha}, \tilde{v}) : \tilde{\alpha} \in [-1, 1] \text{ and } 0 \leqslant \tilde{v} < v_B^r(\hat{\alpha})\}. \tag{24}$$

Thus, $T_\varnothing^r$ is a weighted average between the average score of types in $T_{\varnothing+}^r$ and the average score of types in $T_{\varnothing-}^r$. The former is higher than the latter since all types in $T_{\varnothing+}^r$ have higher scores than those in $T_{\varnothing-}^r$. This implies that $\mathbb{E}_\varnothing^r[v]$ cannot exceed the average score of types in $T_{\varnothing+}^r$ or

$$\mathbb{E}_\varnothing^r[v] \leqslant \frac{\int_{v_B^r(\hat{\alpha})}^{v_A^r(\hat{\alpha})} \tilde{v} dF(\tilde{v})}{F(v_A^r(\hat{\alpha})) - F(v_B^r(\hat{\alpha}))},$$

which can be rewritten as (22). □

We then observe

$$\begin{aligned}
&\frac{u_A^r(\hat{\alpha}; v_A^r(\hat{\alpha}), v_B^r(\hat{\alpha})) - u_B^r(\hat{\alpha}; v_A^r(\hat{\alpha}), v_B^r(\hat{\alpha}))}{1 - F(v_A^r(\hat{\alpha}))} && (25)\\
=&\varepsilon_A(\hat{\alpha}) + q_A + \tau\mathbb{E}_A^r[v] + \frac{F(v_A^r(\hat{\alpha}))}{1 - F(v_A^r(\hat{\alpha}))}\tau\mathbb{E}_\varnothing^r[v]\\
&- \frac{1 - F(v_B^r(\hat{\alpha}))}{1 - F(v_A^r(\hat{\alpha}))}(\varepsilon_B(\hat{\alpha}) + q_B) - \frac{1 - F(v_B^r(\hat{\alpha}))}{1 - F(v_A^r(\hat{\alpha}))}\tau\mathbb{E}_B^r[v] - \frac{F(v_B^r(\hat{\alpha}))}{1 - F(v_A^r(\hat{\alpha}))}\tau\mathbb{E}_\varnothing^r[v]\\
\leqslant&\varepsilon_A(\hat{\alpha}) + q_A + \tau\mathbb{E}_A^r[v] + \frac{F(v_A^r(\hat{\alpha}))}{1 - F(v_A^r(\hat{\alpha}))}\tau\mathbb{E}_\varnothing^r[v]\\
&- (\varepsilon_B(\hat{\alpha}) + q_B) - \frac{1 - F(v_B^r(\hat{\alpha}))}{1 - F(v_A^r(\hat{\alpha}))}\tau\mathbb{E}_B^r[v] - \frac{F(v_B^r(\hat{\alpha}))}{1 - F(v_A^r(\hat{\alpha}))}\tau\mathbb{E}_\varnothing^r[v]\\
=&\hat{\alpha} + \Delta + \tau\mathbb{E}_A^r[v] + \frac{\tau}{1 - F(v_A^r(\hat{\alpha}))}\Big(\big(F(v_A^r(\hat{\alpha})) - F(v_B^r(\hat{\alpha}))\big)\mathbb{E}_\varnothing^r[v] - \big(1 - F(v_B^r(\hat{\alpha}))\big)\mathbb{E}_B^r[v]\Big)
\end{aligned}$$

$$
\begin{aligned}
&\leqslant \hat{\alpha} + \Delta + \tau \mathbb{E}_A^r[v] + \frac{\tau}{1 - F(v_A^r(\hat{\alpha}))}\Big(\int_{v_B^r(\hat{\alpha})}^{v_A^r(\hat{\alpha})} \tilde{v} dF(\tilde{v}) - \int_{v_B^r(\hat{\alpha})}^{1} \tilde{v} dF(\tilde{v}) \Big) \\
&= \hat{\alpha} + \Delta + \tau \mathbb{E}_A^r[v] + \frac{\tau}{1 - F(v_A^r(\hat{\alpha}))}\Big(- \int_{v_A^r(\hat{\alpha})}^{1} \tilde{v} dF(\tilde{v}) \Big) = \hat{\alpha} + \Delta \leqslant 0. \qquad (26)
\end{aligned}
$$

The first equality follows from substituting (13). The first inequality holds since $v_B^r(\hat{\alpha}) \leqslant v_A^r(\hat{\alpha})$ (by Claim 5) and thus $\frac{1-F(v_B^r(\hat{\alpha}))}{1-F(v_A^r(\hat{\alpha}))} \geqslant 1$ and $\varepsilon_B(\hat{\alpha}) + q_B \geqslant 0$. The second inequality follows from Claim 6 and the definition of $\mathbb{E}_B^r[v]$. The last equality follows from the definition of $\mathbb{E}_A^r[v]$. The last inequality holds since $\delta \leqslant 0$ and thus, by (8), $\hat{\alpha} = -\Delta - \tau\hat{\delta} \leqslant -\Delta$ due to the assumption that $\hat{\delta} \geqslant 0$. In sum, we have $u_A^r(\hat{\alpha}; v_A^r(\hat{\alpha}), v_B^r(\hat{\alpha})) - u_B^r(\hat{\alpha}; v_A^r(\hat{\alpha}), v_B^r(\hat{\alpha})) \leqslant 0$, which implies that $\psi(\hat{\alpha}) \geqslant \hat{\alpha}$ since $u_A^r(\cdot; v_A^r(\hat{\alpha}), v_B^r(\hat{\alpha})) - u_B^r(\cdot; v_A^r(\hat{\alpha}), v_B^r(\hat{\alpha}))$ is strictly increasing.

Let $\hat{\alpha}^r \in [\hat{\alpha}, \alpha_0]$ denote a fixed point of ψ that is just shown to exist. Since $v_A^r(\hat{\alpha}) = \hat{v}_A$, $v_A^r(\alpha_0) = \underline{v}$, and $v_A^r(\cdot)$ is decreasing, it must be that $\hat{v}_A^r(\hat{\alpha}^r) \in [\underline{v}, \hat{v}_A]$. We also must have $\hat{v}_B^r(\hat{\alpha}^r) \leqslant \underline{v}$, since otherwise the set $T_A^r \cup T_B^r$ would be a proper subset of $T_A \cup T_B$ while the measure of $T_A^r \cup T_B^r$ must equal $\kappa_A + \kappa_B$, a contradiction. Thus, we have $\hat{v}_A^r(\hat{\alpha}^r) \geqslant \underline{v} \geqslant \hat{v}_B^r(\hat{\alpha}^r)$, which implies $\hat{\delta}^r = \mathbb{E}_A^r[v] - \mathbb{E}_B^r[v] \geqslant 0$ as desired.

Let us next fix an equilibrium with some $\hat{\alpha}^r$, $\hat{v}_A^r$, $\hat{v}_B^r$, and $\hat{\delta}^r \geqslant 0$ under RA. We show that there exists an equilibrium with $\hat{\alpha} \leqslant \hat{\alpha}^r$ and $\hat{\delta} \geqslant 0$ under UA. Assume $\hat{\alpha}^r < 1$ (since otherwise there is nothing to prove). To begin, note that $\delta^r \geqslant 0$ implies $\hat{v}_A^r \geqslant \hat{v}_B^r$. Then, we must have $\hat{v}_A^r \geqslant \underline{v}$ since otherwise we would have $\hat{v}_B^r \leqslant \hat{v}_A^r < \underline{v}$, which implies that the measure of $T_A^r \cup T_B^r$ would exceed $\kappa_A + \kappa_B (= 1 - F(\underline{v}))$. Given $\hat{v}_A^r \geqslant \underline{v}$, we must have $\hat{v}_B^r \leqslant \underline{v}$ since otherwise we would have $\hat{v}_A^r \geqslant \hat{v}_B^r > \underline{v}$, which implies that the measure of $T_A^r \cup T_B^r$ would fall short of $\kappa_A + \kappa_B$ even though $\hat{v}_B^r > 0$. Also, we have $\hat{\alpha}^r \leqslant \alpha_0$ since $1 - G(\hat{\alpha}^r) = \frac{\kappa_A}{1-F(\hat{v}_A^r)} \geqslant \frac{\kappa_A}{1-F(\underline{v})} = 1 - G(\alpha_0)$.

Let us now construct a mapping ξ which maps each $\alpha \in [-1, \alpha_0]$ to some $\alpha' \in [-1, \alpha_0]$ and whose fixed point will correspond to an equilibrium under UA. Define $v_A(\alpha)$ in the same manner as $v_A^r(\alpha)$ above. Define also $\delta(\alpha) = \frac{\kappa_A+\kappa_B}{\kappa_B}\big(e(v_A(\alpha)) - e(\underline{v})\big)$ and note that $\delta(\alpha) \geqslant 0$ for all $\alpha \in [-1, \alpha_0]$ since $v_A(\alpha) \geqslant v_A(\alpha_0) = \underline{v}$ for all $\alpha \in [-1, \alpha_0]$. Then, the mapping is defined as $\xi(\alpha) = \max\{-\Delta - \tau\delta(\alpha), -1\}$. It is straightforward to see that if $\hat{\alpha}$ is a fixed point of ψ, then the threshold type $\hat{\alpha}$ together with the cutoff scores $\hat{v}_A = v_A(\hat{\alpha})$ and $\hat{v}_B = \underline{v}$ can constitute an equilibrium under UA.

Plug now $(\hat{\alpha}^r, \hat{v}_A^r, \hat{v}_B^r)$ instead $(\hat{\alpha}; v_A^r(\hat{\alpha}), v_B^r(\hat{\alpha}))$ into $u_A^r(\cdot)$ and $u_B^r(\cdot)$ in (25). One can then follow the same derivation until the penultimate term in (26) (i.e., $\hat{\alpha}^r + \Delta$) to obtain

$$
0 \leqslant \frac{u_A^r(\hat{\alpha}^r, \hat{v}_A^r, \hat{v}_B^r) - u_B^r(\hat{\alpha}^r, \hat{v}_A^r, \hat{v}_B^r)}{1 - F(\hat{v}_A^r)} \leqslant \hat{\alpha}^r + \Delta,
$$

where the first inequality holds since $(\hat{\alpha}^r, \hat{v}_A^r, \hat{v}_B^r)$ constitutes an equilibrium with $\hat{\alpha}^r < 1$

under RA. Thus, $\hat{\alpha}^r \geqslant -\Delta$. Since $v_A(\hat{\alpha}^r) = \hat{v}_A^r \geqslant \underline{v}$ implies $\delta(\hat{\alpha}^r) \geqslant 0$, we have $\xi(\hat{\alpha}^r) = \max\{-\Delta - \tau\delta(\hat{\alpha}^r), -1\} \leqslant \hat{\alpha}^r$. That $\xi(-1) \geqslant -1$ and $\xi(\hat{\alpha}^r) \leqslant \hat{\alpha}^r$ then implies the existence of a fixed point $\hat{\alpha} \in [-1, \hat{\alpha}^r]$ of the mapping $\xi(\cdot)$, as desired. Also, $\hat{\alpha} \leqslant \hat{\alpha}^r$ implies that the cutoff score for A associated with $\hat{\alpha}$ satisfies $v_A(\hat{\alpha}) \geqslant v_a(\hat{\alpha}^r) \geqslant \underline{v}$, so the equilibrium prestige gap is nonnegative.

Proof of Part (ii) Given the proof of Part (i), it suffices to show that the utilitarian welfare in an equilibrium with $\hat{\alpha}$, $\hat{v}_A$, and $\hat{v}_B = \underline{v}$ under UA is (weakly) lower than that in an equilibrium with $\hat{\alpha}^r \geqslant \hat{\alpha}$, $\hat{v}_A^r \in [\underline{v}, \hat{v}_A]$, and $\hat{v}_B^r \in (0, \underline{v}]$ under RA.

As in the proof of Proposition 1, we consider a hypothetical situation in which the prestige of majors remains the same as in the equilibrium under RA while the student assignments are given as in the equilibrium under UA. This change only affects the utility of students types whose assignments change. To denote those types, we let $T_{jj'}$ denote the set of types who enroll in j under UA and in j' under RA. There are four such sets with positive measures:

$$\begin{aligned}
T_{AB} &= \{(\tilde{\alpha}, \tilde{v}) : \tilde{\alpha} \in [\hat{\alpha}, \hat{\alpha}^r) \text{ and } \tilde{v} \geqslant \hat{v}_A\} \\
T_{BA} &= \{(\tilde{\alpha}, \tilde{v}) : \tilde{\alpha} \geqslant \hat{\alpha}^r \text{ and } \tilde{v} \geqslant [\hat{v}_A^r, \hat{v}_A)\} \\
T_{\varnothing B} &= \{(\tilde{\alpha}, \tilde{v}) : \tilde{\alpha} < \hat{\alpha}^r \text{ and } \tilde{v} \geqslant [\hat{v}_B^r, \underline{v})\} \\
T_{B\varnothing} &= \{(\tilde{\alpha}, \tilde{v}) : \tilde{\alpha} \geqslant \hat{\alpha}^r \text{ and } \tilde{v} \geqslant [\underline{v}, \hat{v}_A^r)\}.
\end{aligned}$$

Clearly, T_{AB} and T_{BA} have the same measure, which is also true for $T_{\varnothing B}$ and $T_{B\varnothing}$ since the seats in B are fully assigned under both UA and RA, given the assumption $\hat{v}_B^r > 0$. The types in $T_{AB} \cup T_{BA}$ become (weakly) worse off under the hypothetical situation than under RA since they prefer their assignments under RA, given that the prestige (and quality) of majors remains the same as in the equilibrium under RA. For the types in $T_{\varnothing B} \cup T_{B\varnothing}$, their aggregate utility from the major quality and prestige (which come from major B) does not change, since the major quality and prestige do not change moving form RA to the hypothetical situation and since $T_{\varnothing B}$ and $T_{B\varnothing}$ have the same measure. Let $\tilde{m}$ denote the measure of $T_{\varnothing B}$ or $T_{B\varnothing}$. Then, the aggregate major fit of the types in $T_{\varnothing B} \cup T_{B\varnothing}$ (weakly) falls going from RA to the hypothetical situation since, under RA, it is equal to

$$\int_{(\tilde{\alpha},\tilde{v}) \in T_{\varnothing B}} \varepsilon_B(\tilde{\alpha}) dG(\tilde{\alpha}) dF(\tilde{v}) \geqslant \varepsilon_B(\hat{\alpha}^r)\tilde{m}$$

while under the hypothetical situation, it is equal to

$$\int_{(\tilde{\alpha},\tilde{v}) \in T_{B\varnothing}} \varepsilon_B(\tilde{\alpha}) dG(\tilde{\alpha}) dF(\tilde{v}) \leqslant \varepsilon_B(\hat{\alpha}^r)\tilde{m},$$

where the inequalities hold since $\varepsilon(\cdot)$ is decreasing. In sum, the utilitarian welfare (weakly) falls going from the equilibrium under RA to the hypothetical situation. Let us now move from the hypothetical situation to the equilibrium under UA. This movement only involves the changes in the prestige utilities, which does not affect the utilitarian welfare. Thus, the proof is complete. □

B.4.2 Condition for major B to be fully enrolled under RA

In this section we provide a condition for major B to be fully enrolled under RA. More specifically, the lemma below provides a sufficient condition under which $\hat{v}^r_B > 0$ in equilibrium (a condition assumed in Part (ii) of Proposition 5).

Lemma 4. *Suppose that*

$$\big(1 - F(v_0)\big)\big(\varepsilon_A(\alpha_0) + q_A\big) < \varepsilon_B(\alpha_0) + q_B, \tag{27}$$

where (α_0, v_0) *satisfies*

$$G(\alpha_0) = \kappa_B \tag{28}$$

$$\big(1 - G(\alpha_0)\big)\big(1 - F(v_0)\big) = \kappa_A. \tag{29}$$

Then, every equilibrium under RA has $\hat{v}^r_B > 0$ *so the major* B *fills its capacity.*

Proof. Suppose for contradiction that (27) holds but an equilibrium with $\hat{v}^r_B = 0$ exists under RA. Then, by (18), we have $G(\hat{\alpha}^r) \leqslant \kappa_B$, which implies $\hat{\alpha}^r \leqslant \alpha_0$ and $\hat{v}^r_A \geqslant v_0$ due to the definition of α_0 and v_0 in (28) and (29). Observe next that $T_\varnothing = \{(\tilde{\alpha}, \tilde{v}) : \tilde{\alpha} \geqslant \hat{\alpha}^r \text{ and } \tilde{v} < \hat{v}^r_A\}$. Given this and $T_A = \{(\tilde{\alpha}, \tilde{v}) : \tilde{\alpha} \geqslant \hat{\alpha}^r \text{ and } \tilde{v} \geqslant \hat{v}^r_A\}$, we have

$$\big(1 - F(\hat{v}^r_A)\big)\mathbb{E}^r_A[v] + F(\hat{v}^r_A)\mathbb{E}^r_\varnothing[v] = \mathbb{E}[v]. \tag{30}$$

Also, given that $\hat{v}^r_B = 0$, we have $\mathbb{E}^r_B[v] = e(\hat{v}^r_B) = \mathbb{E}[v]$ and thus, by (13), $u_B(\alpha; \hat{v}^r_A, \hat{v}^r_B) = \varepsilon_B(\alpha) + q_B$. It also follows from (13) and (30) that $u_A(\alpha; \hat{v}^r_A, \hat{v}^r_B) = \big(1 - F(\hat{v}^r_A)\big)(\varepsilon_A(\alpha) + q_A)$. Then, by (27),

$$\begin{aligned} u_A(\hat{\alpha}^r; \hat{v}^r_A, \hat{v}^r_B) &= \big(1 - F(\hat{v}^r_A)\big)\big(\varepsilon_A(\hat{\alpha}^r) + q_A\big) \\ &\leqslant \big(1 - F(v_0)\big)\big(\varepsilon_A(\alpha_0) + q_A\big) \\ &< \varepsilon_B(\alpha_0) + q_B \leqslant \varepsilon_B(\hat{\alpha}^r) + q_B = u_B(\hat{\alpha}^r; \hat{v}^r_A, \hat{v}^r_B), \end{aligned}$$

where the weak inequalities follow from the facts that $\hat{\alpha}^r \leqslant \alpha_0$ and $\hat{v}^r_A \geqslant v_0$ and that $\varepsilon_A(\cdot)$ is increasing while $\varepsilon_B(\cdot)$ is decreasing. This equation means that the type $\hat{\alpha}^r$ strictly prefers applying to B, a contradiction. □

B.4.3 Comparison between UA and RA

Assume that $v \sim U[0,1]$ and $\alpha \sim U[-1,1]$. Let $q = 1.5$, $q_A = q + \frac{1}{2}\Delta$, and $q_A = q - \frac{1}{2}\Delta$ with $\Delta \in [0,3]$. The graphs in Figure B.4 show that the comparisons in Proposition 5 hold true unless

Δ is so high. However, if Δ is sufficiently high, then the cutoff score at B under RA falls to zero, which means that B fails to fill its quota (since too many students apply to A), as shown in the lower left graph of Figure B.4. When this happens, the prestige gap between the two majors becomes higher under RA, as shown in the upper left graph. This negatively affects the utilitarian welfare under RA so that it falls below that under UA, as shown in the lower right graph.

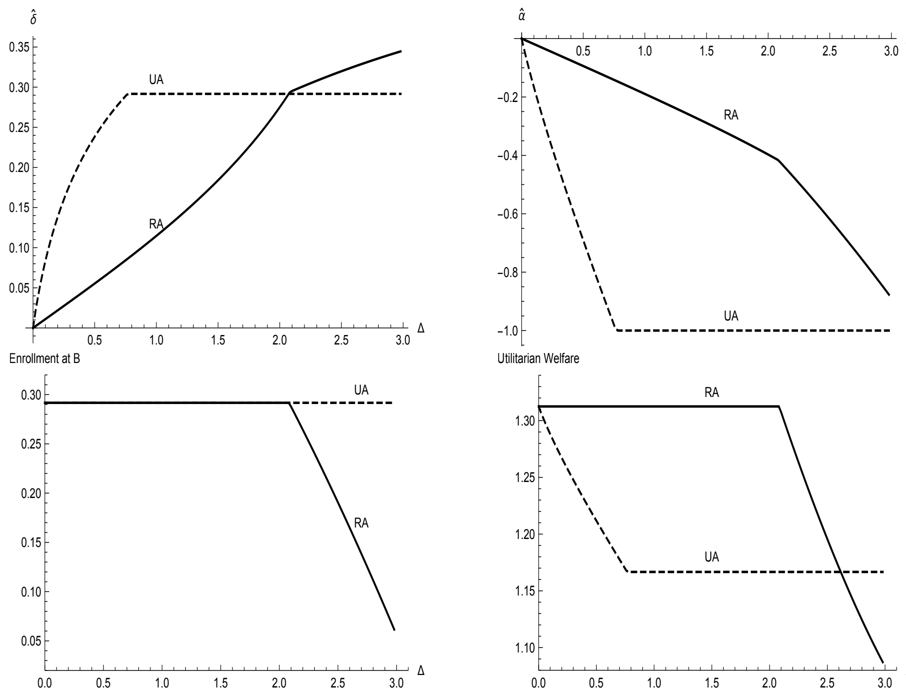


Figure B.4: Comparison between UA and RA.